



education

Department:
Education

PROVINCE OF KWAZULU-NATAL

**Grades 8 and 9
Just-in-Time Training Workshop
2019: No.1**

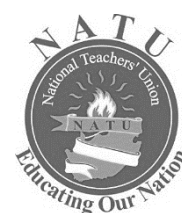
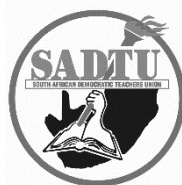
Facilitator's Guide

Mathematics

Endorsed by:



Jika iMfundo
what I do matters



WORKSHOP PROGRAM – GRADES 8 and 9
(on page 1 of the Participant's Guide)

Session 1: 2 hours	<p>DEVELOPING FRACTION CONCEPTS</p> <p>1.1 Engaging with the Intermediate and Senior Phase CAPS documents Use Appendix 1 to list the Grades where each fraction concept is introduced and taught.</p> <p>1.2 Three different models used to develop the concept of fractions Use Appendix 2 to describe the three models and to give examples of these models.</p> <p>1.3 Five meanings of fractions Use Appendix 3 to investigate the five meanings of fractions.</p> <p>1.4 Types of fraction problems Use Appendix 4 to write a short description of the Grade 4, Grade 5 and Grade 6 problems given for the Type of fraction allocated to you. Make up your own Grade 7, 8 and 9 problems, similar to the Grade 5 and 6 problems you have been allocated. Share them with the class.</p>
Session 2: 1 hour 30 minutes	<p>USING CONCRETE APPARATUS TO TEACH FRACTIONS</p> <p>2.1 What is meant by concrete, semi-concrete and abstract methods of teaching? Write down what you understand by these terms. These terms are then explained using Appendix 5</p> <p>2.2 Using Concrete and Semi-Concrete Material to teach the multiplication and division of fractions The following topics are covered</p> <ol style="list-style-type: none"> a) Multiplication of fractions b) Division of fractions c) Using abstract reasoning to explain why we change division to multiplication and invert the second number
Session 3: 2 hours	<p>FRACTION EXTENSION ACTIVITIES</p> <p>Groups work on given questions and write their solutions on flipchart paper which they stick on the wall. The participants move around the round studying the solutions, writing down the answers and asking the group member for assistance if they need it.</p>

NOTE TO THE FACILITATOR FOR SESSION 1

(allow 2 hours for this session)

RESOURCES:

- Each person will need a copy of the Participant's Handout and Resources Material
- Each pair will need one sheet of flipchart paper and a marker pen

PRE-TEST

30 minutes	<i>Each educator writes the test on the question paper. Take the papers in.</i>
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ORGANISE THE EDUCATORS INTO GROUPS

Before you start, make sure the Participants are sitting in groups

To form groups:

- STEP 1:** *Decide on the size of your group (5 or 6 in a group is a good size)*
- STEP 2:** *Mentally divide the number of Participants in your group by 5 (or 6).
Suppose you have 30 Participants. $30 \div 5 = 6$. So, you will have 6 groups of 5.*
- STEP 3:** *Because there are going to be 6 groups, go around the class, numbering the Participants from 1 to 6.
Tell the Participants to remember their numbers (NOTE: they are bad at this!)*
- STEP 4:** *Tell the 1s to form a group, the 2s to form a group, and so on.
If they have remembered their numbers, you should have 6 groups of equal size.
If you don't have groups of equal size, move Participants around to get equal-sized groups.*

1.1: ENGAGING WITH THE INTERMEDIATE AND SENIOR PHASE CAPS

25 minutes	<i>Working with their group, the Participants use Appendix 1 given in the Resource Handout to list the Grades where each fraction concept is introduced and taught.</i> <ul style="list-style-type: none"> • <i>Discuss the answers with the Participants.</i>
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1.2: THREE DIFFERENT MODELS USED TO DEVELOP THE CONCEPT OF FRACTIONS

25 minutes	<i>Working with their group, the Participants use Appendix 2 given in the Resource Handout to investigate the 3 models listed in the Intermediate Phase CAPS and discuss why these 3 models are not given in the Senior Phase CAPS.</i> <ul style="list-style-type: none"> • <i>Discuss the answers with the Participants.</i>
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1.3: FIVE MEANINGS OF FRACTIONS

25 minutes	<i>Working with their group, the Participants use Appendix 3 given in the Resource Handout to investigate five different meanings of fractions as given in an article found on the internet. Three of these meanings are relevant to the Foundation and Intermediate Phases and two are relevant to the Senior Phase.</i> <ul style="list-style-type: none"> • <i>Discuss the answers with the Participants.</i>
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1.4: TYPES OF FRACTION PROBLEMS

45 minutes	<i>Divide the class into 8 groups. Allocate one of the types of fraction problems to each group. The groups have to first give a short description of the problems given for Grade 4, 5 and 6. The groups then make up Grades 7, 8 and 9 examples similar to the Grade 4, 5 and 6 ones. The examples are stuck up around the classroom and the class moves around interrogating 'the person on duty'.</i> <ul style="list-style-type: none"> • <i>Discuss the examples with the Participants.</i>
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SESSION 1: DEVELOPING FRACTION CONCEPTS

1.1. Engaging with the Intermediate and Senior Phase CAPS documents (p 2)



Work with your group

Use APPENDIX 1 (on pages 1 to 3 in the Resources Handout) to answer the following questions.

- 1) Both the Intermediate and Senior Phase CAPS documents list, for each grade, the concepts that are different to the previous grade.

What concepts are new to the learners in

- a) Grade 4?

Sevenths

Addition of fractions with the same denominators

- b) Grade 5?

Ninths, tenths, elevenths and twelfths

Counting in fractions

Subtraction of fractions with the same denominators

Addition and subtraction of mixed numbers

Fractions of whole numbers with whole number answers

- c) Grade 6?

They name, order and compare all common fractions

Tenths and hundredths

Addition and subtraction of fractions where one denominator is a multiple of another

Fractions of whole numbers with fractional answers

Decimals

Percentages

Equivalence between fractions, decimals and percentages

- d) Grade 7?

Hundredths

Multiplication of common fractions

Percentage of a whole

Percentage increase and decrease

e) Grade 8?

Division of common fractions

Squares, cubes, square roots and cube roots of common fractions

f) Grade 9?

Learners consolidate number knowledge and calculation techniques for common fractions

- 2) In which Grades do the learners ‘Compare and Order Fractions’? **Grades 4 to 7**
- 3) Study the references to Addition and Subtraction in the Phase Overview. In which Grades do the learners
- Start adding fractions with the same denominators? **Grade 4**
 - Start subtracting fractions with the same denominator? **Grade 5**
 - Start adding and subtracting fractions where one denominator is a multiple of another? **Grade 6**
 - Start adding and subtracting fractions where one denominator is NOT a multiple of the other? **Grade 7**
 - Start working with the addition and subtraction of mixed numbers? **Grade 5**
- 4) Study the references to the Equivalence of Division and Fractions
- What do we mean by the equivalence of Division and Fractions? **That $1 \div 5 = \frac{1}{5}$**
 - In which Grades do the learners ‘recognise, describe and use the equivalence of division and fractions’? **Grades 4 and 5**
- 5) Study the references to Multiplication and Division in the Phase Overview. In which Grades do the learners
- Find the fractions of whole numbers which result in whole number answers? **Grades 5 to 9**
 - Find the fractions of whole numbers which do not necessarily result in whole number answers? **Grades 6 to 9**
 - Multiply common fractions and mixed numbers? **Grades 7, 8 and 9**
 - Divide fractions? **Grades 8 and 9**
- 6) In which Grades do the learners calculate the squares, cubes, square roots and cube roots of common fractions? **Grades 8 and 9**
- 7) In which Grades do the learners solve problems in contexts involving fractions? **Grades 4 to 9**

- 8) Study the references to Percentages. In which Grade do the learners
- a) Start working with percentages? **Grade 6**
 - b) Find the percentages of whole numbers? **Grades 6 to 9**
 - c) Calculate the percentage of parts of a whole? **Grades 7 to 9**
 - d) Calculate percentage increase or decrease? **Grades 7 to 9**
 - e) Calculate amounts if given percentage increase or decrease? **Grades 8 and 9**
 - f) Solve problems in contexts involving percentages? **Grades 8 and 9**
- 9) Study the references to Equivalent Forms in the Phase Overview. In which Grades do the learners
- a) Recognise and use equivalent forms of common fractions where one denominator is a multiple of another? **Grades 4 to 9**
 - b) Recognise the equivalence between common fractions, decimal fractions and percentage forms of the same number? **Grades 6 to 9**

1.2. Three different models used to develop the concept of fractions (p 5)



Work with your group

Use APPENDIX 2 (on pages 4 to 14 in the Resources Handout) to answer the following questions.

- 1) The Clarification Notes for Grade 4, for Grade 5 and for Grade 6 CAPS state that learners should work with apparatus and diagrams to develop different ways of thinking about fractions. These different models are
- Region or Area Models
 - Length or Measurement Models
 - Set Models

- a) Explain the difference between Region Models, Length Models and Set Models. Give examples of each model.

Region or Area Models are objects that are divided into equal parts. They develop the concept of fractions as part of a whole and also fractions as a measure.

Examples of area models: circles cut into fraction pieces or diagrams of pies, rectangles or other geometric shapes divided into fraction pieces (paper folding), fractions using square or dotted paper, geoboards

Length or Measurement Models that are divided along a straight line. They also develop the concept of fractions as part of a whole and also fractions as a measure.

Examples of length models include fraction strips, Cuisenaire rods and number lines

Set models consist of a collection of objects. They can form the basis of thinking about a fraction of a number e.g. $\frac{1}{3}$ of 12.

Examples of set models include counters of any kind in different arrangements

- b) Why does the CAPS document say that the learners should not only work with one kind of fraction model?

Working with one kind of fraction model can limit their understanding of fractions.

- c) What does the CAPS document suggest when using fraction models in the Intermediate Phase?
- The CAPS document suggests that all three models should be used when dealing with each fractions concept in the Intermediate Phase.***

- 2) Use the Clarification Notes for Grades 7, 8 and 9 CAPS in Appendix 2 to answer the following question.

The Senior Phase CAPS does not specify that these three models should be used to develop the concept of fractions

- a) Why do you think the three models are not specified?

The Senior Phase states that the focus is on context free calculations which means that it is not necessary to use diagrams.

- b) Does this mean that the three models should not be used in the Senior Phase?

It is assumed that the learners fraction concepts have been developed to such a stage that using diagrams to illustrate fraction calculations should not be necessary. HOWEVER, all three types of diagrams should be used where a learner is still struggling with fractions.

In the Senior Phase the learners solve problems in contexts, and one of the problem-solving techniques they could use is 'drawing a diagram'. So they could use the three fraction models to help them solve problems.

1.3 Five meanings of fractions (p 7)



Work with your group

Use APPENDIX 3 (on pages 15 to 17 in the Resources Handout) to answer the following questions.

- 1) This article suggests that three meanings of fractions should be developed in the Foundation and Intermediate Phase.
- a) Give a brief summary of these three meanings of fractions.

The cut or part of a whole meaning involves the imagined or actual action of cutting a whole number into equal parts. This could involve cutting a quantity according to its length, area, volume or mass. For example, you could cut a piece of string into 4 equal parts; each part is $\frac{1}{4}$ of the whole; you could cut a circle into halves; you could cut a loaf of bread into 8 parts having the same volume; each part is $\frac{1}{8}$ of the volume of the loaf of bread

The part of a group or set meaning involves selecting objects from a group. For example, if 14 of 23 books are novels, then we know that $\frac{14}{23}$ of the books are novels.

The name for a point meaning involves associating marks on measuring devices with fraction names. It is also called the measure meaning of fractions. For example, a number line marked in quarters can be used as a ruler for measuring length using fractional parts of the unit of length

- b) How do these meanings compare to the three different models that the Intermediate Phase CAPS says should be used to develop the concept of fractions (given in Appendix 2)?

The three models in Appendix 3 are very similar to the three models in Appendix 2. The order is just different.

2) It also suggests that the last two meanings of fractions should be developed in the Senior Phase.

a) Give as brief summary of these two meanings of fractions

The ratio meaning involves the comparison between two quantities. $\frac{2}{3}$ implies the ratio of 2 is to 3.

Indicated division meaning refers to the fact that a fraction is another way of dividing numbers

b) Give one topic where Meaning 4 would be used in the Senior Phase?

The ratio meaning would be covered when the Senior Phase learners deal with Ratio and Proportion.

c) Give one topic where Meaning 5 would be covered in the Senior Phase?

The indicated division meaning would be covered when the Senior Phase learners convert between fractions and decimals.

1.4 Types of Fraction Problems (p 9)



Divide the class into 8 groups.

Use APPENDIX 4 (on pages 18 in the Resources Handout) to answer the following questions.

Your facilitator will give each group one of the different types of problems to work with.

(Type 9 only appears in Grade 6 and deals with calculations only, so we will leave Type 9 out of this exercise)

Learners from Grade 4 to Grade 9 have to solve fraction problems in contexts. The Intermediate Phase CAPS Document lists different Types of problems that should be dealt with in Grade 4, 5 and 6.

- 1) For your allocated Type, write a short description of the problems given for Grade 4, Grade 5 and Grade 6.

TYPE 1: whole is a single object

- ***GRADE 4: one fraction of a chocolate bar; subtraction***
- ***GRADE 5: two fractions of a chocolate cake; addition and subtraction***
- ***GRADE 6: one fraction of a chocolate bar; subtraction and division***

TYPE 2: whole is a collection of objects

- ***GRADE 4: sharing of chocolates where one chocolate has to be cut up***
- ***GRADE 5: find the fraction of wall panels; subtraction***
- ***GRADE 6: fraction of the number of hours; addition and subtraction***

TYPE 3: relationship

- ***GRADE 4: find a unit fraction of an amount of money that is divisible by the denominator***
- ***GRADE 5: find a unit fraction of an amount of money that is not divisible by the denominator***
- ***GRADE 6: find a non-unit fraction of an amount of money***

TYPE 4: ratio

- ***GRADE 4: given the amount of milk needed for one batch of biscuits, find the amount needed for 5 batches***
- ***GRADE 5: given the ratio of sugar to butter for one batch, use the ratio to find the amount of butter needed***
- ***GRADE 6: given the amount of milk needed for 40 biscuits, determine the amount of milk needed for 1 000 biscuits***

TYPE 5: comparator

- ***GRADE 4: comparing two fractions of a quantity***
- ***GRADE 5: comparing two equivalent fractions of a quantity***
- ***GRADE 6: comparing fractions of a metre using division by 100 and***

TYPE 6: unit of measurement

- ***GRADE 4: subtracting fractions of a metre where both fractions have the same denominator***
- ***GRADE 5: repeated subtraction of fractions of a metre where both fractions have the same denominator***
- ***GRADE 6: repeated subtraction of fractions of a metre where the fractions have different denominators***

TYPE 7: number

- **GRADE 4:** find a fraction between a mixed number and a whole number
- **GRADE 5:** find a fraction between two mixed numbers where the fractions have different denominators
- **GRADE 6:** placing fractions and mixed numbers with different denominators on a number line

TYPE 8: fractional parts put together to make a whole

- **GRADE 4:** given a simple fraction of an orange given to one athlete (here $\frac{1}{2}$), find out how many is needed for 55 athletes
- **GRADE 5:** given the fraction of a bottle of cool drink given to one person, find out how much is needed for 35 children
- **GRADE 6:** given the fraction of a bottle of cool drink and fraction of a bar of chocolate given to one person, find out how much is needed for 500 children

- 2) For your allocated Type, make up your own Grade 7, 8 and 9 examples similar to the three given. Use Appendix 1 and Appendix 2 to make sure that the number range and operations asked for in your example are suitable for Grades 7, 8 and 9 .

Write your examples on flip-chart paper and stick the paper up.

Be prepared to explain to your colleagues how your examples meets the requirements for Grade 7, 8 and 9.

This means that you need to be able to explain how it fits into the Type allocated to you and also how where it fits into the Grades 7, 8 and 9 fraction curriculum.

Answers will differ.

*One person from the pair remains at their examples and is the person 'on duty'.
The rest of the class walks around looking at the answers and asking questions of the person 'on duty'.
The person on duty must be prepared to explain their examples to whoever asks and must also be able to explain where the example it fits in the Gr 7, 8 and 9 fraction curriculum.*

NOTE TO THE FACILITATOR FOR SESSION 2

(allow 90 minutes (1½ hours) for this session)

RESOURCES

- Each participant needs 2 sheets of scrap paper

2.1: WHAT IS MEANT BY CONCRETE, SEMI-CONCRETE AND ABSTRACT METHODS OF TEACHING?

<i>30 minutes</i>	<p><i>The participants write down what they understand by concrete, semi-concrete and abstract methods of teaching fractions.</i></p> <p><i>The facilitator uses Appendix 5 (on page 20 in the Resources Handout) to explain to the Participants the difference between concrete, semi-concrete and abstract phases of teaching.</i></p> <p><i>The participants write down whether their understanding of the terms has changed or not.</i></p>
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2.2: USING CONCRETE AND SEMI-CONCRETE MATERIAL TO TEACH THE MULTIPLICATION AND DIVISION OF FRACTIONS

<i>20 minutes</i>	<p>a) Multiplication of Fractions</p> <p><i>Example 1: Use paper folding to determine $\frac{1}{2} \times \frac{1}{4}$</i></p> <p><i>Example 2: Draw a model to determine $\frac{1}{2} \times \frac{2}{3}$</i></p> <p><i>Complete Activity 1 where the Participants have to interpret multiplication diagrams and draw their own multiplication diagrams</i></p>
<i>20 minutes</i>	<p>b) Division of Fractions</p> <p><i>Discuss the 5 examples which use diagrams to explain division</i></p> <p><i>Complete Activity 2 where the Participants draw diagrams to work out the answer to division examples</i></p>
<i>20 minutes</i>	<p>c) Using Abstract Reasoning to explain why we change division to multiplication and invert the second number</p> <p><i>Discuss reciprocals and the meaning of division</i></p> <p><i>Example 3: Use mathematical methods to divide by a fraction</i></p> <p><i>Complete Activity 3 where the Participants find the answers to the division of fractions by writing down every step and then analyse where 4 learners went wrong and then give the correct answer.</i></p>

SESSION 2: USING CONCRETE APPARATUS TO TEACH FRACTIONS

2.1 Concrete, Semi-Concrete and Abstract Methods of Teaching Fractions (p 11)



Answer question 1) with your group.
Then listen to your facilitator who will read the information given in APPENDIX 5 (on page 20 in the Resources Handout) to the class.
Then answer question 2) with your group.

- 1) Before listening to the reading of Appendix 5, write down what do you understand by
 - a) Concrete methods of teaching fractions
Answers will differ
 - b) Semi-concrete methods of teaching fractions
Answers will differ
 - c) Abstract methods of teaching fractions
Answers will differ

- 2) After listening to the reading of Appendix 5, write down whether your understanding has changed or not. Did you have any misconceptions about the terms ‘concrete’, semi-concrete’ and ‘abstract’?

2.2 Using concrete material to teach the multiplication and division of fractions

NOTE:

Multiplication and division are dealt with in the following grades

- a) In Grade 5 the learners find fractions of whole numbers which result in whole number answers.
- b) In Grade 6 the learners find fractions of whole numbers
- c) In Grade 7 the learners
 - find fractions of whole numbers
 - multiply common fractions including mixed numbers.
- d) In Grade 8 the learners
 - find fractions of whole numbers
 - multiply common fractions including mixed numbers
 - divide whole numbers and common fractions by common fractions.
- e) In Grade 9 the learners use all four operations with common fractions and mixed numbers.

a) MULTIPLICATION OF FRACTIONS (p 12)



The Facilitator and the participants use paper folding to illustrate a concrete method of explaining the multiplication of fractions.

You then use squared paper to illustrate a semi-concrete method of explaining the multiplication of fractions.

The Participants work on Activity 1 with a partner, and this is marked.

Learners need to understand the *conceptual ideas behind multiplication of fractions*, not simply learn the rules to follow (i.e. multiply the numerators, multiply the denominators). Using models to represent the process will help them to better understand these concepts and increase their ability to remember the rules.

- ✓ Learners know that $5 \times 7 = 35$ because 5 groups of seven is 35.
- ✓ An example like " $3 \times \frac{1}{2}$ " is easy to understand as $3 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3\frac{1}{2}$
- ✓ When confronted with the calculation $\frac{2}{3} \times \frac{1}{4}$, most learners have difficulty understanding that the product of these numbers will be smaller than either of them. This is an expected reaction because their previous experience of multiplication (e.g. 4×3) makes them expect that the product will be bigger than either of the factors.

EXAMPLE 1 (Using a concrete model)

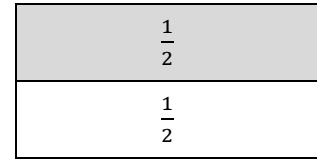
Use paper folding to determine $\frac{1}{2} \times \frac{1}{4}$, remembering that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{2}$ of $\frac{1}{4}$

Solution

STEP 1:

Use paper folding to determine $\frac{1}{2}$ of the piece of paper.

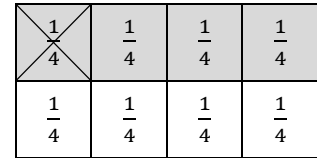
Shade in the required area



STEP 2:

Use paper folding to determine $\frac{1}{4}$ of $\frac{1}{2}$ of the sheet of paper by taking the same piece of paper and fold it in quarters (fourths) in the other direction.

Shade in $\frac{1}{4}$ of $\frac{1}{2}$ using another colour or by drawing a cross.



STEP 3:

Count up the number of equal parts (8)

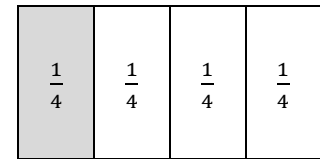
Count up the number of parts that are both shaded grey **and** have a cross in them (1)

$$\therefore \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

NOTE:

Because we know that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2}$ (multiplication is commutative)

We could first find $\frac{1}{4}$ of the piece of paper



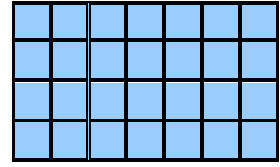
And then find $\frac{1}{2}$ of $\frac{1}{4}$ by using another colour or by drawing a cross.



NOTE:

You know that the area of a rectangle is found by multiplying the length of the one side by the length of the other side.

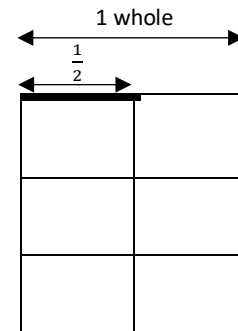
So, the area of this rectangle is 28 squares because the one side is 4 units and the other side is 7 units.

**EXAMPLE 2 (Drawing a Model)**

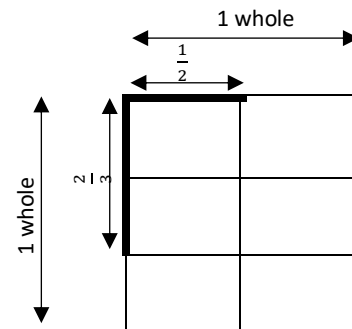
Determine $\frac{1}{2} \times \frac{2}{3}$ using the fact that *area of a rectangle = length of the rectangle \times breadth of a rectangle* and using squared paper

Solution

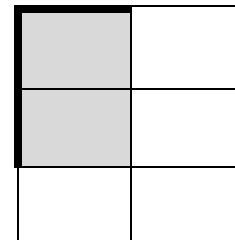
- ✓ To work out $\frac{1}{2} \times \frac{2}{3}$ start off by drawing a rectangle that is 2 units by 3 units on squared paper. (These measurements come from the denominators of the two fractions). The fractions being multiplied represent length of the sides, whereas the answer represents area.
- ✓ Follow the following procedure:
 - Divide the side that is 2 units long into halves



- Divide the side that is 3 units long into thirds



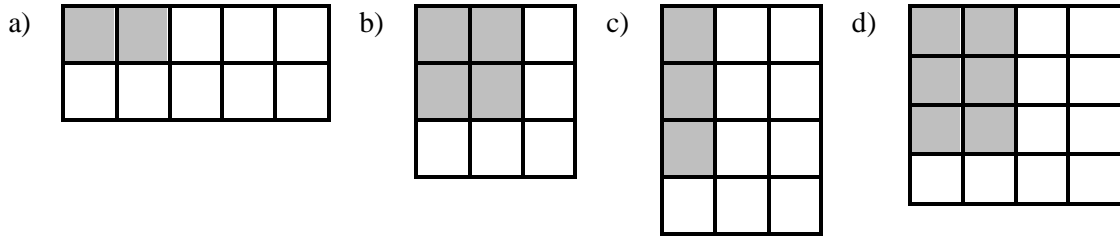
- Shade in the area that is $\frac{1}{2}$ by $\frac{2}{3}$



From the diagram we can see that $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

ACTIVITY 1

1) The pictures show fractional areas. Write down the multiplication sentence that goes with each one. Give the answer in simplest form.



SOLUTION

a) $\frac{2}{5} \times \frac{1}{2} = \frac{2}{10} = \frac{1}{5}$

b) $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

c) $\frac{1}{3} \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$

d) $\frac{3}{4} \times \frac{2}{4} = \frac{6}{16} = \frac{3}{8}$

2) Draw diagrams as in the previous question to determine the answers to the following:

a) $\frac{1}{4} \times \frac{1}{2}$ b) $\frac{1}{5} \times \frac{1}{3}$ c) $\frac{5}{8} \times \frac{1}{6}$ d) $\frac{3}{4} \times \frac{5}{6}$

SOLUTION

$\frac{1}{4} \times \frac{1}{2}$ $= \frac{1}{8}$	$\frac{1}{5} \times \frac{1}{3}$ $= \frac{1}{15}$	$\frac{5}{8} \times \frac{1}{6} = \frac{5}{48}$	$\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8}$

b) DIVISION OF FRACTIONS (p 16)



The Facilitator discusses Example 3 which illustrates semi-concrete methods of explaining the division of fraction with the Participants.
The Participants work on Activity 2 with a partner and this is marked.

- ✓ To divide means to split into **equal** parts or groups.
- ✓ Before learners learn to *divide fractions*, they should first explore the division of whole number by a fraction and only then go onto the division of a fraction by a fraction. But, teaching learners how to divide with fractions by simply showing them how to multiply the dividend (the first number) and the reciprocal of the divisor (the second number) offers practically no insight to their mathematical understanding because no connections have been made to what they already know about division.

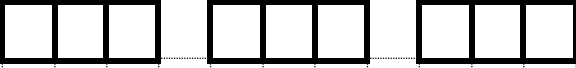

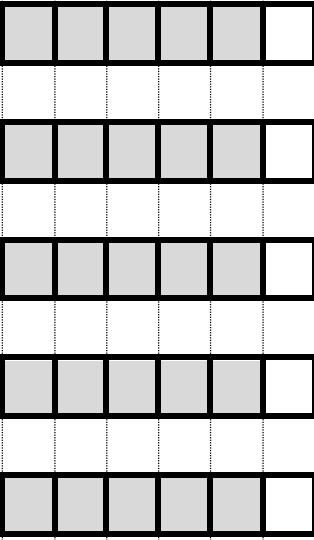
EXAMPLE 3 (drawing models)	
<p>a) How many halves are there in 4? From the diagram it can be seen that there are eight halves in 4. We write this $4 \div \frac{1}{2} = 8$</p>	
<p>b) $2 \div \frac{1}{5}$ means ‘how many fifths are there in 2’ We take 2 wholes and count up the number of fifths There are 10 fifths in 2 wholes, so $2 \div \frac{1}{5} = 10$. We write $2 \div \frac{1}{5} = 2 \times 5 = 10$</p>	
<p>c) $2 \div \frac{2}{5}$ means ‘how many two-fifths are there in 2’ We take 2 wholes and count up the number of two-fifths There are 5 two-fifths in 2 wholes, so $2 \div \frac{2}{5} = 5$ We write $2 \div \frac{2}{5} = \frac{2}{1} \times \frac{5}{2} = \frac{10}{2} = 5$</p>	
<p>d) We can use a diagram to find the answer to $\frac{1}{2} \div \frac{1}{6}$ $\frac{1}{2} \div \frac{1}{6}$ is really asking ‘how many sixths are there in $\frac{1}{2}$’? There are 3 sixths in one-half, so $\frac{1}{2} \div \frac{1}{6} = 3$ We write $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1}$ $= \frac{6}{2}$ $= 3$</p>	<p>Look at the pizzas below: How many sixths of a pizza fit into $\frac{1}{2}$ a pizza? How many in ? There are 3 sixths in $\frac{1}{2}$ a pizza</p>
<p>e) How many $\frac{2}{3}$ are there in $2\frac{2}{3}$? From the diagram it can be seen that there are four two-thirds in $2\frac{2}{3}$. We write $2\frac{2}{3} \div \frac{2}{3} = \frac{8}{3} \times \frac{3}{2} = \frac{24}{6} = 4$</p>	<p>We write this $2\frac{2}{3} \div \frac{2}{3} = 4$</p>

ACTIVITY 2

Draw diagrams to work out

- 1) How many thirds there are in 3?
- 2) $2 \div \frac{1}{6}$
- 3) $5 \div \frac{5}{6}$

SOLUTION

1)		<i>There are 9 thirds in 3</i>
2)		$2 \div \frac{1}{6} = 12$
3)		$5 \div \frac{5}{6} = 5 \text{ lots of } \frac{5}{6} \text{ plus } 5 \text{ lots of } \frac{1}{6} = 5 \text{ lots of } \frac{5}{6} + 1 \text{ lot of } \frac{5}{6} = 6$

c) **USING ABSTRACT REASONING TO EXPLAIN WHY WE CHANGE DIVISION TO MULTIPLICATION AND INVERT THE SECOND NUMBER (p 18)**



The Facilitator illustrates abstract methods of explaining the division of fractions to the Participants.

The Participants work on Activity 3 with a partner and this is marked

- ✓ The learners need to understand reciprocals before they start to use them to divide fractions.

Reciprocals

Two numbers are reciprocals of each other if their product is 1. In other words, when you multiply them, you get 1.

- $\frac{4}{3}$ is a reciprocal of $\frac{3}{4}$ because $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$
- $\frac{7}{1}$ is a reciprocal of $\frac{1}{7}$ because $\frac{1}{7} \times \frac{7}{1} = \frac{7}{7} = 1$
- $\frac{1}{7}$ is a reciprocal of $\frac{7}{1}$ because $\frac{7}{1} \times \frac{1}{7} = \frac{7}{7} = 1$

- ✓ Dividing by a whole number can be thought of as dividing or sharing something between a certain number of people.

☞ If you divide by two, you cut something in half.

Then (for example) half of a half can be found by cutting the half into halves. The size of half of a half can be given in relation to the original whole.

☞ The following generalisation can be made:

- Dividing a number by 2 is the same as finding $\frac{1}{2}$ of the number.
- Dividing a number by 3 is the same as finding $\frac{1}{3}$ of the number.

☞ The algorithm the learners will have to learn ultimately is, instead of dividing by a fraction, multiply by its reciprocal number.

- Instead of dividing by $\frac{1}{2}$, multiply by its reciprocal 2.
- Instead of dividing by $\frac{3}{4}$, multiply by its reciprocal $\frac{4}{3}$.
- Instead of dividing by $\frac{5}{8}$, multiply by its reciprocal $\frac{8}{5}$.

EXAMPLE 4 (Why the division algorithm works)

Determine a) $\frac{1}{2} \div 2$

b) $\frac{3}{8} \div \frac{2}{5}$

Solution

a) $\frac{1}{2} \div 2 = \frac{\frac{1}{2}}{2}$

$$= \frac{\frac{1}{2}}{2} \times \frac{\frac{1}{2}}{\frac{1}{2}}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{2}{1} \times \frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{2}$$

$$= \frac{1}{4}$$

$$= \frac{1}{4}$$

... In the first step write the division as a fraction

... We want to change the denominator into 1. We do this by multiplying by 1 or by $\frac{\frac{1}{2}}{\frac{1}{2}}$

... Multiply both the numerator and denominator by $\frac{1}{2}$

... Simplify

This is the same as multiplying by the inverse of the denominator: $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

b) $\frac{3}{8} \div \frac{2}{5} = \frac{\frac{3}{8}}{\frac{2}{5}}$

$$= \frac{\frac{3}{8}}{\frac{2}{5}} \times \frac{\frac{5}{2}}{\frac{5}{2}}$$

$$= \frac{\frac{3}{8} \times \frac{5}{2}}{\frac{2}{5} \times \frac{5}{2}}$$

$$= \frac{15}{16}$$

$$= \frac{15}{16}$$

$$= \frac{15}{16}$$

... Write the division as a fraction

... Multiply by 1. This time $1 = \frac{\frac{5}{2}}{\frac{5}{2}}$ because $\frac{5}{2}$ is the reciprocal of the $\frac{2}{5}$ in the denominator

This is the same as multiplying by the inverse of the denominator: $\frac{3}{8} \div \frac{2}{5} = \frac{3}{8} \times \frac{5}{2} = \frac{15}{16}$

- ✓ Learners who use the detailed method recognise that the top fraction is being multiplied by the reciprocal of the bottom fraction. The process of discovering this ‘trick’ for themselves gives them ownership of the larger conceptual understanding. In other words, learners who learn to divide fractions in this way understand at a fundamental level *why* the trick works.

ACTIVITY 3

1) Do the following calculations like Example 3 *showing your solutions in full*:

a) $\frac{12}{25} \div \frac{21}{50}$

$$\text{SOLUTION: } \frac{12}{25} \div \frac{21}{50} = \frac{12}{25} \times \frac{50}{21} = \frac{12 \times 50}{25 \times 21} = \frac{8}{7} = \frac{8}{7}$$

b) $3\frac{2}{5} \div \frac{1}{3}$

$$\text{SOLUTION: } 3\frac{2}{5} \div \frac{1}{3} = \frac{17}{5} \times \frac{3}{1} = \frac{17 \times 3}{5 \times 1} = \frac{51}{5} = 10\frac{1}{5}$$

c) $4\frac{1}{2} \div 3\frac{1}{3}$

$$\text{SOLUTION: } 4\frac{1}{2} \div 3\frac{1}{3} = \frac{9}{2} \times \frac{3}{10} = \frac{9 \times 3}{2 \times 10} = \frac{27}{20} = 1\frac{7}{20}$$

ACTIVITY 3 (continued)

- 2) Look at the working presented by the following learners. In each case:
- Explain the mistake made by the learner
 - Give the correct solution to each calculation.

<p><i>Peter: I worked out that</i></p> $\frac{1}{8} \div \frac{2}{5} = \frac{2}{40}$ $= \frac{1}{20}$	<p><i>Jabu: My calculation gives</i></p> $\frac{3}{4} \div 1\frac{1}{4} = \frac{3}{5} \times \frac{4}{1}$ $= \frac{12}{5}$ $= 2\frac{2}{5}$
<p><i>Thabo: How did I go wrong?</i></p> $1\frac{1}{10} \div 2\frac{1}{4} = \frac{11}{10} \times \frac{9}{4}$ $= \frac{99}{40}$	<p><i>Romy: Why doesn't this work?</i></p> $10\frac{1}{8} \div 2\frac{1}{4} = 10 \div 2 + \frac{1}{8} \div \frac{1}{4}$ $= 5\frac{1}{2}$

SOLUTION**Peter**

- a) *Peter changed the division to multiplication but didn't invert the second fraction*

b) $\frac{1}{8} \div \frac{2}{5} = \frac{1}{8} \times \frac{5}{2} = \frac{5}{16}$

Jabu

- a) *Jabu added 1 to the 4 in the denominator of $\frac{3}{4}$ to get $\frac{3}{5}$ and changed the division to multiplication and inverted the $\frac{1}{4}$ only*

b) $\frac{3}{4} \div 1\frac{1}{4} = \frac{3}{4} \div \frac{5}{4} = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

Thabo

- a) *Thabo multiplied the two mixed numbers and didn't divide them. (The 2nd number wasn't inverted)*

b) $1\frac{1}{10} \div 2\frac{1}{4} = \frac{11}{10} \div \frac{9}{4} = \frac{11}{10} \times \frac{4}{9} = \frac{22}{45}$

Romy

- a) *Romy divided the whole numbers and the fractions separately*

b) $10\frac{1}{8} \div 2\frac{1}{4} = \frac{81}{8} \div \frac{9}{4} = \frac{81}{8} \times \frac{4}{9} = \frac{9}{2} = 4\frac{1}{2}$

NOTE TO THE FACILITATOR FOR SESSION 3

(allow 2 hours for this session)

RESOURCES: Flipchart paper and pens
Prestik / Bostick

10 minutes	<p><u>STEP 1</u> Divide the group of participants into 4 approximately equal sized groups. Each group should get 2 or 3 sheets of flipchart paper and at least one pen.</p>
1 hour	<p><u>STEP 2</u> Allocate the questions as given on the list on the next page. The groups work on their questions and write the answers on flipchart paper</p>
40 minutes	<p><u>STEP 3</u> All four groups stick their sheets of paper on the wall. One member of the group must stay with the answers and must be prepared to explain their working out to the rest of the class. The rest of the class walk around copying down the other groups' answers, and asking the group member standing at the answers if they don't understand anything</p>
10 minutes	<p><u>STEP 4</u> Discuss with the whole class any examples where problems were experienced</p>

SESSION 3: FRACTION EXTENSION ACTIVITIES (p 22)



The class needs to be divided into four groups.

Work with your group on the following questions:

GROUP 1: Numbers 1, 4, 8, 13, 20, 23a, 24

GROUP 2: Numbers 2, 5, 10, 14, 16, 21, 23b, 24

GROUP 3: Numbers 3, 6, 11, 12, 17, 22, 23c, 24

GROUP 4: Numbers 7, 9, 15, 18, 19, 23d, 24

NOTE TO THE FACILITATORS: *I went through the 24 questions and decided all four groups should do 23 and 24. I then went through the rest of the questions, decided which ones I thought were easy, which ones were moderate, and which ones were difficult, and shared them out.*

Most KZN schools should either be setting Problem Solving Papers or should be including Problem Solving activities in their tests and examinations.

In order to develop problem solving abilities, Grade 8 and 9 learners need to spend time on Extension Activities that are not just “word problems”. In other words, it should not always be clear which operation should be used to arrive at the answer. Here we give you some extension activities to help the learners develop their problem-solving skills.

For each question:

- a) Determine the answers and write the answers on flipchart paper.
You may use your calculator to find the answers.

Show all working out

- b) Say which grade the question is most suitable for

EXTENSION QUESTIONS

- 1) I am equivalent to
- $\frac{1}{2}$
- .

The sum of my numerator and denominator is 18.
What fraction am I?

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \dots$$

$$6 + 12 = 18$$

So, I am $\frac{6}{12}$

HINT: Find some fractions that are equivalent to $\frac{1}{2}$ and find the sum of their numerator and denominator until you find the fraction.

- 2) I am equivalent to
- $\frac{3}{5}$
- .

The product of my numerator and denominator is 135.
What fraction am I?

$$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25} = \frac{18}{30} = \frac{7}{14} = \dots$$

$$9 \times 15 = 135$$

So, I am $\frac{9}{15}$

- 3) I am equivalent to
- $\frac{3}{4}$
- .

The difference between my numerator and denominator is 8.
What fraction am I?

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \frac{21}{28} = \frac{24}{32} = \frac{27}{36} = \dots$$

$$32 - 24 = 8$$

So, I am $\frac{24}{32}$

- 4) My numerator is a single-digit odd number
My denominator is a two-digit palindrome.
If you add 1 to both my numerator and denominator, you get a fraction equivalent to $\frac{1}{2}$
What am I?

My numerator could be 1; 3; 5; 7 or 9

My denominator could be 11; 22; 33; 44; 55; 66; 77; 88; or 99

Fractions equivalent to $\frac{1}{2}$ are: $\frac{2}{4}$; $\frac{3}{6}$; $\frac{4}{8}$; $\frac{5}{10}$; $\frac{6}{12}$; $\frac{7}{14}$; ...

Subtracting 1 from these numerators and denominators,

We get: $\frac{0}{1}$; $\frac{1}{3}$; $\frac{2}{5}$; $\frac{3}{7}$; $\frac{4}{9}$; $\frac{5}{11}$; $\frac{6}{13}$; $\frac{7}{15}$; $\frac{8}{17}$; $\frac{9}{19}$; $\frac{11}{21}$; ...

The only fraction that satisfies the requirements is $\frac{5}{11}$

NOTE: A number that is a **palindrome** reads the same backwards and forward, for example the number 10201.

- 5) I am a unit fraction
 My denominator is a multiple of 4
 I am greater than $\frac{1}{9}$
 My denominator has more than 3 factors
 What am I?

NOTE: All **unit fractions** have 1 in their numerator.

I could be: $\frac{1}{4}; \frac{1}{8}; \frac{1}{12}; \frac{1}{16}; \frac{1}{20}; \frac{1}{24}; \dots$

Looking at the denominators: 4 has 3 factors; 8 has 4 factors ✓; 12 has 6 factors ✓; ...

$$\frac{1}{9} = 0,111\ 111\ 111 \dots$$

$$\frac{1}{8} = 0,125 > \frac{1}{9};$$

$$\frac{1}{12} = 0,083\ 333\ 333 \dots < \frac{1}{9}$$

So, I am $\frac{1}{8}$

- 6) Which of the fractions $\frac{3}{5}$, $\frac{7}{10}$ and $\frac{8}{15}$ is closest to $\frac{1}{2}$.

$$\frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10} = \frac{3}{30}$$

$$\text{OR } \frac{3}{5} - \frac{1}{2} = 0,6 - 0,5 = 0,1$$

$$\frac{7}{10} - \frac{1}{2} = \frac{7-5}{10} = \frac{2}{10} = \frac{6}{30}$$

$$\text{OR } \frac{7}{10} - \frac{1}{2} = 0,7 - 0,5 = 0,2$$

$$\frac{8}{15} - \frac{1}{2} = \frac{16-15}{30} = \frac{1}{30} \dots$$

$$\text{OR } \frac{8}{15} - \frac{1}{2} = 0,533\ 333 - 0,5 = 0,033\ 333$$

So, $\frac{8}{15}$ is closest to $\frac{1}{2}$

- 7) Gladys, Kim and Justin worked on a mathematical problem and came up with the answers $3\frac{1}{2}$, $3\frac{2}{3}$ and $3\frac{3}{5}$ respectively. The correct answer was $3\frac{7}{12}$. Which of the three learners was closest to the correct answer?

$$\text{Gladys: } 3\frac{7}{12} - 3\frac{1}{2} = \frac{43}{12} - \frac{7}{2} = \frac{43-42}{12} = \frac{1}{12} = \frac{5}{60}$$

$$\text{OR } 3\frac{7}{12} - 3\frac{1}{2} = 3,583\ 333 \dots - 3,5 = 0,083\ 333 \dots$$

$$\text{Kim: } 3\frac{7}{12} - 3\frac{2}{3} = \frac{43}{12} - \frac{11}{3} = \frac{43-44}{12} = -\frac{1}{12} = -\frac{5}{60}$$

$$\text{OR } 3\frac{7}{12} - 3\frac{2}{3} = 3,583\ 333 \dots - 3,666\ 666 \dots = -0,083\ 333 \dots$$

$$\text{Justin: } 3\frac{7}{12} - 3\frac{3}{5} = \frac{43}{12} - \frac{18}{5} = \frac{215-216}{60} = -\frac{1}{60} \dots$$

$$\text{OR } 3\frac{7}{12} - 3\frac{3}{5} = 3,583\ 333 \dots - 3,06 = -0,016\ 666$$

$3\frac{3}{5}$ is closest to $3\frac{7}{12}$

So, Justin was closest.

- 8) The numbers in this set form a pattern.

What mixed number is the next term in the sequence $0; \frac{3}{8}; \frac{3}{4}; 1\frac{1}{8}; 1\frac{1}{2}; 1\frac{7}{8}; 2\frac{1}{4}$?

$$\frac{3}{8} - 0 = \frac{3}{8}$$

$$\frac{3}{4} - \frac{3}{8} = \frac{6-3}{8} = \frac{3}{8}$$

So, $\frac{3}{8}$ is added to each number to get the next number

$$2\frac{1}{4} + \frac{3}{8} = \frac{9}{4} + \frac{3}{8} = \frac{18+3}{8} = \frac{21}{8} = 2\frac{5}{8}$$

So, the next number is $2\frac{5}{8}$

- 9)

a) Write the next three numbers in the pattern $5\frac{3}{8}; 4\frac{1}{2}; 3\frac{5}{8}$

b) What rule is used to make the pattern?

$$4\frac{1}{2} - 5\frac{3}{8} = \frac{9}{2} - \frac{43}{8} = \frac{36}{8} - \frac{43}{8} = -\frac{7}{8}$$

$$3\frac{5}{8} - 4\frac{1}{2} = \frac{28}{8} - \frac{9}{2} = \frac{28}{8} - \frac{36}{8} = -\frac{7}{8}$$

So, $\frac{7}{8}$ is subtracted from each number to get the next number

$$3\frac{5}{8} - \frac{7}{8} = \frac{29}{8} - \frac{7}{8} = \frac{22}{8} = 2\frac{3}{4}$$

$$2\frac{3}{4} - \frac{7}{8} = \frac{11}{4} - \frac{7}{8} = \frac{22}{8} - \frac{7}{8} = \frac{15}{8} = 1\frac{7}{8}$$

$$1\frac{7}{8} - \frac{7}{8} = \frac{15}{8} - \frac{7}{8} = \frac{8}{8} = 1$$

a) The next three numbers are $2\frac{3}{4}; 1\frac{7}{8}; 1$

b) The rule is 'subtract $\frac{7}{8}$ '

- 10) Is the product of a whole number and a proper fraction *always greater than or equal to* one of the two factors? Give examples to explain your answer.

$$1 \times \frac{1}{2} = \frac{1}{2} \quad \text{the answer is less than 1 but equal } \frac{1}{2} \quad \text{correct}$$

$$6 \times \frac{5}{6} = \frac{6}{1} \times \frac{5}{6} = 5 \quad \text{the answer is less than 6 but greater than } \frac{5}{6} \quad \text{correct}$$

$$75 \times \frac{9}{11} = 61\frac{4}{11} \quad \text{the answer is less than 75 but greater than } \frac{9}{11} \quad \text{correct}$$

From looking at these three examples, we can see that the answer is always greater than or equal to the proper fraction, but less than the whole number.

- 11) A recipe calls for $1\frac{1}{4}$ cups of flour and $1\frac{3}{4}$ cups of milk.

If you only have a $\frac{1}{4}$ cup measuring cup, how could you use it to measure the flour and the milk?

$$1\frac{1}{4} = \frac{5}{4} \quad \text{For } 1\frac{1}{4} \text{ cups of flour, we need } 5 \times \frac{1}{4} \text{ cups of flour}$$

$$1\frac{3}{4} = \frac{7}{4} \quad \text{For } 1\frac{3}{4} \text{ cups of milk, we need } 7 \times \frac{1}{4} \text{ cups of milk}$$

12) Compare using < or > or =:

- a) $4 \times \frac{1}{5} \square 4 \times \frac{1}{10}$
 b) $15 \div \frac{1}{9} \square 15 \div \frac{7}{9}$
 c) $\frac{1}{12} \times 32 \square 32 \times \frac{1}{12}$
 d) $\frac{4}{15} \div 8 \square 8 \div \frac{4}{15}$
 e) $9 \div \frac{1}{4} \square 12 \div \frac{1}{3}$
 f) $\frac{1}{5} \times \frac{6}{7} \square \frac{1}{2} \times \frac{6}{7}$

SOLUTION

$$\begin{aligned} \text{a) } 4 \times \frac{1}{5} &= \frac{4}{5} = \frac{8}{10} \\ 4 \times \frac{1}{10} &= \frac{4}{10} \\ \frac{8}{10} &> \frac{4}{10} \text{ SO } 4 \times \frac{1}{5} > 4 \times \frac{1}{10} \end{aligned}$$

$$\begin{aligned} \text{b) } 15 \div \frac{1}{9} &= \frac{15}{1} \times \frac{9}{1} = 135 \\ 15 \div \frac{7}{9} &= \frac{15}{1} \times \frac{9}{7} = \frac{135}{7} = 19\frac{2}{7} \\ 135 &> 19\frac{2}{7} \text{ SO } 15 \div \frac{1}{9} > 15 \div \frac{7}{9} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{1}{12} \times 32 &= \frac{8}{3} \\ 32 \times \frac{1}{12} &= \frac{8}{3} \\ \text{SO } \frac{1}{12} \times 32 &= 32 \times \frac{1}{12} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{4}{15} \div 8 &= \frac{4}{15} \times \frac{1}{8} = \frac{1}{30} \\ 8 \div \frac{4}{15} &= \frac{8}{1} \times \frac{15}{4} = 30 \\ \text{SO } \frac{4}{15} \div 8 &< 8 \div \frac{4}{15} \end{aligned}$$

$$\begin{aligned} \text{e) } 9 \div \frac{1}{4} &= \frac{9}{1} \times \frac{4}{1} = 36 \\ 12 \div \frac{1}{3} &= \frac{12}{1} \times \frac{3}{1} = 36 \\ \text{SO } 9 \div \frac{1}{4} &= 12 \div \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{f) } \frac{1}{5} \times \frac{6}{7} &= \frac{6}{35} \\ \frac{1}{2} \times \frac{6}{7} &= \frac{3}{7} = \frac{15}{35} \\ \frac{1}{5} \times \frac{6}{7} &< \frac{1}{2} \times \frac{6}{7} \end{aligned}$$

13) Amanda likes making Banana Muffins.

- a) She plans to make muffins for her friends but decides to only make 6 muffins.

Work out the correct amounts for each of the ingredients.

1 cup flour

$1\frac{1}{2}$ teaspoon baking powder

$\frac{1}{4}$ teaspoon salt

$\frac{3}{8}$ cup sugar

$\frac{1}{2}$ banana

$\frac{1}{2}$ cup milk

$\frac{1}{2}$ egg

$\frac{1}{8}$ cup oil

Banana Muffins

Makes 12 muffins

2 cups flour

3 teaspoons baking powder

$\frac{1}{2}$ teaspoon salt

$\frac{3}{4}$ cup sugar

1 banana

1 cup milk

1 egg

$\frac{1}{4}$ cup oil



- b) She then decides to make 30 muffins for all the learners in her class.
Work out the correct amounts for each of the ingredients.

30 muffins = (12 + 12 + 6) muffins = $2\frac{1}{2}$ times the recipe

$$2\frac{1}{2} \times 2 = \frac{5}{2} \times \frac{2}{1} = 5 \text{ cups flour}$$

$$2\frac{1}{2} \times 3 = \frac{5}{2} \times \frac{3}{1} = \frac{15}{2} = 7\frac{1}{2} \text{ teaspoons baking powder}$$

$$2\frac{1}{2} \times \frac{1}{2} = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4} = 1\frac{1}{4} \text{ teaspoons salt}$$

$$2\frac{1}{2} \times \frac{3}{4} = \frac{5}{2} \times \frac{3}{4} = \frac{15}{8} = 2\frac{1}{8} \text{ cup sugar}$$

$$2\frac{1}{2} \text{ bananas}$$

$$2\frac{1}{2} \times 1 = 2\frac{1}{2} \text{ cup milk}$$

$$2\frac{1}{2} \text{ eggs}$$

$$2\frac{1}{2} \times \frac{1}{4} = \frac{5}{2} \times \frac{1}{4} = \frac{5}{8} \text{ cup oil}$$

- 14) Which of these cards gives an answer that is different from the other two?
Show all your working.

A

$20 \div \frac{4}{15}$

B

$63 \div \frac{7}{8}$

C

$36 \div \frac{12}{25}$

A: $20 \div \frac{4}{15} = \frac{20}{1} \times \frac{15}{4} = 75$

B: $63 \div \frac{7}{8} = \frac{63}{1} \times \frac{8}{7} = 72$

C: $36 \div \frac{12}{25} = \frac{36}{1} \times \frac{25}{12} = 75$

So, B is different to A and C

- 15) You know that $1\frac{2}{3} \times 3\frac{4}{5} = 3\frac{4}{5} \times 1\frac{2}{3}$.

Is $1\frac{2}{3} \div 3\frac{4}{5} = 3\frac{4}{5} \div 1\frac{2}{3}$? Explain your answer.

$$1\frac{2}{3} \div 3\frac{4}{5} = \frac{5}{3} \div \frac{19}{5} = \frac{5}{3} \times \frac{5}{19} = \frac{25}{57}$$

$$3\frac{4}{5} \div 1\frac{2}{3} = \frac{19}{5} \div \frac{5}{3} = \frac{19}{5} \times \frac{3}{5} = \frac{57}{25}$$

So, $1\frac{2}{3} \div 3\frac{4}{5} \neq 3\frac{4}{5} \div 1\frac{2}{3}$

We know that they are not equal because division (of any numbers) is not commutative.

We know that multiplication is always commutative. That is why $1\frac{2}{3} \times 3\frac{4}{5} = 3\frac{4}{5} \times 1\frac{2}{3}$.

- 16) Which expression has a smaller quotient: $6\frac{1}{2} \div \frac{1}{2}$ or $6\frac{1}{2} \div 3\frac{1}{2}$?

$$6\frac{1}{2} \div \frac{1}{2} = \frac{13}{2} \times \frac{2}{1} = \frac{13}{1} = 13$$

$$6\frac{1}{2} \div 3\frac{1}{2} = \frac{13}{2} \div \frac{7}{2} = \frac{13}{2} \times \frac{2}{7} = \frac{13}{7} = 1\frac{6}{7}$$

So, $6\frac{1}{2} \div 3\frac{1}{2}$ has a smaller quotient than $6\frac{1}{2} \div \frac{1}{2}$

- 17) What number divided by $\frac{2}{3}$ is 24? Check your answer.

$$\square \div \frac{2}{3} = 24, \text{ so } 24 \times \frac{2}{3} = \frac{24}{1} \times \frac{2}{3} = \frac{16}{1} = 16$$

The number is 16

CHECK: $16 \div \frac{2}{3} = \frac{16}{1} \times \frac{3}{2} = \frac{24}{1} = 24 \checkmark$

- 18) Write down a division equation using $\frac{3}{8}$, 2 and $5\frac{1}{3}$

$$2 \div \frac{3}{8} = 5\frac{1}{3}$$

$$\left(\frac{2}{1} \times \frac{8}{3} = \frac{16}{3} = 5\frac{1}{3}\right)$$

OR

$$2 \div 5\frac{1}{3} = \frac{3}{8}$$

$$\left(2 \div \frac{16}{3} = \frac{2}{1} \times \frac{3}{16} = \frac{3}{8}\right)$$

- 19) A recipe for pasta salad calls for $2\frac{2}{3}$ cups of cooked pasta.

Thuli has 5 cups of cooked pasta.

Does Thuli have enough pasta to double the recipe for pasta salad?

To double the recipe Thuli will need $2 \times 2\frac{2}{3} = \frac{2}{1} \times \frac{8}{3} = \frac{16}{3} = 5\frac{1}{3}$ cups of cooked pasta

So, she doesn't have enough pasta.

- 20) If you change one of the digits in $2\frac{1}{4} + 3\frac{5}{8}$ to a 9, and you want to get the **smallest** possible answer, which digit should you change?

Work methodically, changing each digit in turn to 9 and either write each answer with the same denominator or as a decimal

- *Change the 2 to 9: $9\frac{1}{4} + 3\frac{5}{8} = \frac{37}{4} + \frac{29}{8} = \frac{74+29}{8} = \frac{103}{8} = 12,875$*
- *Change the 1 to 9: $2\frac{9}{4} + 3\frac{5}{8} = \frac{17}{4} + \frac{29}{8} = \frac{34+29}{8} = \frac{63}{8} = 7,875$*
- *Change the 4 to 9: $2\frac{1}{9} + 3\frac{5}{8} = \frac{19}{9} + \frac{29}{8} = \frac{152 + 261}{72} = \frac{413}{72} = 5,73611... \checkmark$*
- *Change the 3 to 9: $2\frac{1}{4} + 9\frac{5}{8} = \frac{9}{4} + \frac{77}{8} = \frac{18+77}{8} = \frac{95}{8} = 11,875$*
- *Change the 5 to 9: $2\frac{1}{4} + 3\frac{9}{8} = \frac{9}{4} + \frac{33}{8} = \frac{18+33}{8} = \frac{51}{8} = 6,375$*
- *Change the 8 to 9: $2\frac{1}{4} + 3\frac{5}{9} = \frac{9}{4} + \frac{32}{9} = \frac{81 + 128}{36} = \frac{209}{36} = 5,8055...$*

The smallest possible answer is $2\frac{1}{9} + 3\frac{5}{8}$

- 21) The difference between two fractions with unlike denominators is $\frac{1}{6}$.

The denominator of the greater fraction is an even number less than 8. Its numerator is 5.

The denominator of the smaller fraction is an odd number and is half the denominator of the greater fraction.

What are the fractions?

Possible first fractions: $\frac{5}{2}; \frac{5}{4}; \frac{5}{6}$

The difference between the first fraction and the second fraction is $\frac{1}{6}$

So, first fraction – second fraction = $\frac{1}{6}$

OR, first fraction – $\frac{1}{6}$ = second fraction

Use (first fraction – $\frac{1}{6}$ = second fraction) to find possible second fractions:

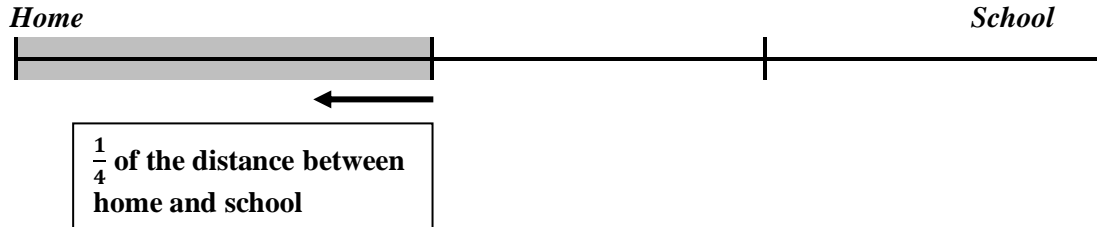
$$\frac{5}{2} - \frac{1}{6} = \frac{15}{6} - \frac{1}{6} = \frac{14}{6} = \frac{7}{3} \quad \text{denominator is odd but is NOT half of 2 } \times$$

$$\frac{5}{4} - \frac{1}{6} = \frac{15}{12} - \frac{2}{12} = \frac{13}{12} \quad \text{denominator is even and NOT half of 4 } \times$$

$$\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \quad \text{denominator is odd and half of 6 } \checkmark$$

The fractions are $\frac{5}{6}$ and $\frac{2}{3}$

- 22) Dennis had walked $\frac{1}{3}$ of the way to school when he realised that he had dropped a book. He turned around and had covered a $\frac{1}{4}$ of the distance between home and school before finding it. What fraction of the total distance between home and school did Dennis now have to walk?



Dennis walked $\frac{1}{3}$ of the way to school. He still had to walk $\frac{2}{3}$ of the way to school

He turned around and walked $\frac{1}{4}$ of the distance between home and school before he found his book.

After he finds this book, he has to walk back along this $\frac{1}{4}$ of the distance plus the $\frac{2}{3}$ of the distance.

So, Dennis still has walk $\frac{1}{4}$ of the distance + $\frac{2}{3}$ of the distance

$$= \frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12} \text{ of the distance}$$

Worked Example:

Using the digits 1, 2, 3, 4, 5 and 6 once only, find two ways to fill the blanks in $\frac{\square}{\square} + \frac{\square\square}{\square\square} = 1$

Solution

- i) Let's start with $\frac{1}{6}$. We know that $\frac{1}{6} + \frac{5}{6} = 1$
- We need to use the digits 2, 3, 4 and 5 to make a fraction equivalent to $\frac{5}{6}$
 - Either list all the fractions equivalent to $\frac{5}{6}$ until you find the correct fraction:

$$\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = \frac{35}{42}$$
 - OR, use trial and error to determine that $\frac{35}{42} = \frac{5}{6}$
 - So, $\frac{1}{6} + \frac{35}{42} = 1$
- ii) Now let's start with $\frac{2}{6} = \frac{1}{3}$. We know that $\frac{2}{6} + \frac{4}{6} = 1$ OR $\frac{1}{3} + \frac{2}{3} = 1$
- We need to use the digits 1, 2, 4 and 5 to find a fraction equivalent to $\frac{4}{6}$ or $\frac{2}{3}$
 - Either list all the fractions equivalent to $\frac{2}{3}$ until you find the correct fractions, or use trial and error to determine that $\frac{34}{51} = \frac{2}{3} = \frac{4}{6}$
 - So, $\frac{2}{6} + \frac{34}{51} = 1$

We could carry on like this until we have found all the possible combinations of numbers.

- 23) For each of the following, the digits 1 to 8 are used only once to find two fractions whose sum is 1.
Find the second fraction given the first fraction.
Use the method given in the worked example on the previous page to assist you.

a) $\frac{17}{34} + \frac{\square\square}{\square\square} = 1$ (1, 3, 4 and 7 have been used, so find the second fraction using the numbers 2, 5, 6 and 8)

$$\frac{17}{34} = \frac{1}{2} \text{ and } \frac{1}{2} + \frac{1}{2} = 1$$

We need to use 2, 5, 6 and 8 to make $\frac{1}{2}$

- Either list all the fractions equivalent to $\frac{1}{2}$ until you find the correct fraction, or use trial and error to determine that $\frac{28}{56} = \frac{1}{2}$

So $\frac{17}{34} + \frac{28}{56} = 1$

b) $\frac{27}{54} + \frac{\square\square}{\square\square} = 1$ (2, 4, 5 and 7 have been used, so find the second fraction using the numbers 1, 3, 6 and 8)

$$\frac{27}{54} = \frac{1}{2} \text{ and } \frac{1}{2} + \frac{1}{2} = 1$$

We need to use 1, 3, 6 and 8 to make $\frac{1}{2}$

- Either list all the fractions equivalent to $\frac{1}{2}$ until you find the correct fraction, or use trial and error to determine that $\frac{18}{36} = \frac{1}{2}$

So, $\frac{27}{54} + \frac{18}{36} = 1$

c) $\frac{26}{78} + \frac{\square\square}{\square\square} = 1$ (2, 6, 7 and 8 have been used, so find the second fraction using the numbers 1, 3, 4 and 5)

$$\frac{26}{78} = \frac{1}{3} \text{ and } \frac{1}{3} + \frac{2}{3} = 1$$

We need to use 1, 3, 4 and 5 to make $\frac{2}{3}$

- Either list all the fractions equivalent to $\frac{2}{3}$ until you find the correct fraction, or use trial and error to determine that $\frac{34}{51} = \frac{2}{3}$

So, $\frac{26}{78} + \frac{34}{51} = 1$

d) $\frac{36}{54} + \frac{\square\square}{\square\square} = 1$ (3, 4, 5 and 6 have been used, so find the second fraction using the numbers 1, 6, 7 and 8)

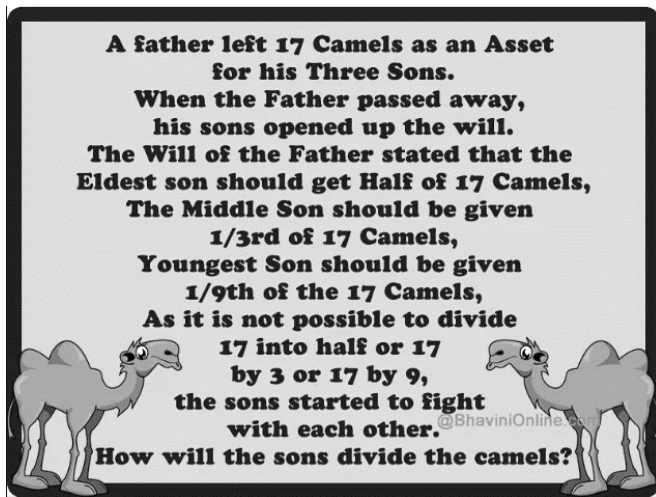
$$\frac{36}{54} = \frac{2}{3} \text{ and } \frac{2}{3} + \frac{1}{3} = 1$$

We need to use 1, 2, 7 and 8 to make $\frac{1}{3}$

- Either list all the fractions equivalent to $\frac{1}{3}$ until you find the correct fraction, or use trial and error to determine that $\frac{27}{81} = \frac{1}{3}$

So, $\frac{36}{54} + \frac{27}{81} = 1$

24)



In desperation, the sons visited a wise woman in the neighbouring village.

After hearing about the dispute, the woman said that she would reflect on it and advise them the next day.

The next morning, the woman told the sons she would give them her own camel, in hopes that it could help them resolve their problem and end the feud.

The sons were puzzled, but pleased to have an additional camel, and began to walk home. On the long walk home, they calculated how a herd of 18 camels might be divided.

- a) There were 17 camels. The sons need to take $\frac{1}{2}$ of the camels, $\frac{1}{3}$ of the camels and $\frac{1}{9}$ of the camels. Calculate $\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$. What do you notice about the answer?

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{9} = \frac{9}{18} + \frac{6}{18} + \frac{2}{18} = \frac{17}{18}$$

The total is not 1

- b) Now calculate $\frac{1}{2}$ of 17 camels + $\frac{1}{3}$ of 17 camels + $\frac{1}{9}$ of 17 camels. What do you notice about the answer?

$$\begin{aligned} & \frac{1}{2} \text{ of } 17 \text{ camels} + \frac{1}{3} \text{ of } 17 \text{ camels} + \frac{1}{9} \text{ of } 17 \text{ camels} \\ &= 8\frac{1}{2} \text{ camels} + 5\frac{2}{3} \text{ camels} + 1\frac{8}{9} \text{ camels} \end{aligned}$$

We end up with fractions of camels – which is impossible

- c) The wise woman gave them 1 camel, meaning that they had 18 camels. Calculate $\frac{1}{2}$ of 18 camels + $\frac{1}{3}$ of 18 camels + $\frac{1}{9}$ of 18 camels. Explain why the one camel helped them divide the number of camels.

$$\begin{aligned} & \frac{1}{2} \text{ of } 18 \text{ camels} + \frac{1}{3} \text{ of } 18 \text{ camels} + \frac{1}{9} \text{ of } 18 \text{ camels} \\ &= 9 \text{ camels} + 6 \text{ camels} + 2 \text{ camels} \\ &= 17 \text{ camels} \end{aligned}$$

After the division of the 18 camels, they had 1 camel left over which they could return to the wise woman.

17 is a prime number so is not divisible by 2, 3 and 9 meaning that the answers were mixed numbers.

18 is divisible by 2, 3 and 9, meaning that the answers were whole numbers.

But, because $\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$ is not equal to 1, there was one camel left over which they could return to the wise woman.