PARTICIPANT'S HANDOUT

2017 TERM 1 TRAINING WORKSHOP

MATHEMATICS

GRADES 8-9

PILO

IMPROVING LEARNER PERFORMANCE

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Just-in-Time Training Workshop Term 1 2017

Participant's Manual

Grade 8-9
Mathematics

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what I do matters

Endorsed by:

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Table of Contents

Unit 1. Algebraic expressions and equations-Grade 8

Objectives

After you have completed this unit you should be able to:

- Recognize and interpret rules or relationships represented in symbolic form
- Identify variables and constants in given formulae and/or equations
- Recognize and identify conventions for writing algebraic expressions
- Identify and classify like and unlike terms in algebraic expressions
- Recognize and identify coefficients and exponents in algebraic expressions

1.1 Expanding and simplifying algebraic expressions

An expression is made up of constants and variables, linked by operational signs and does not include an equal sign. For example $2y + 4$ is an expression. We can use commutative, associative and distributive laws for rational numbers and laws of exponents to:

- add and subtract like terms in algebraic expressions
- multiply integers and monomials by:
  - ✓ monomials $[e.g.\ 2x(2y) = 4xy]$
  - ✓ binomials $\left[e.g.\ 2x\left(2x + \frac{1}{2}\right) = 4x^2 + x\right]$
  - ✓ trinomials $\left[e.g.\ 2x\left(3x^2 + \frac{3}{2}x + 10\right) = 6x^3 + 3x^2 + 20x\right]$

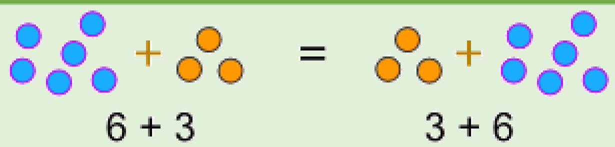
1.1.1 Commutative law

The "Commutative Law" says we can **swap positions of numbers** over and still get the same answer when we **add** or multiply.

Addition

$$a + b = b + a$$

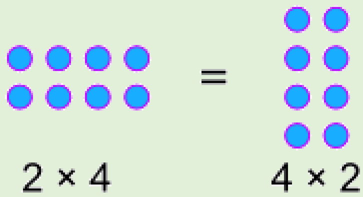
Example 1.1 Distributive law



Multiplication

$$a \times b = b \times a$$

Example 1.2 Distributive law



1.1.2 Distributive law

Numbers and variables satisfy an important general property, called the distributive law. The distributive law says that:

$$a(b + c) = ab + ac$$

In fact, more than this is true:

$$a(b + c + d) = ab + ac + ad$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

and

$$a(b + c + d + e) = ab + ac + ad + ae$$

Example 1.3

We use the distributive law to write expressions such as $3(x + 1)$ in a form that has no brackets,

$$\begin{aligned} 3(x + 1) &= (3 \times x) + (3 \times 1) \\ &= 3x + 3 \end{aligned}$$

We say that we have expanded $3(x + 1)$ to get $3x + 3$. To expand is to get rid of the brackets.

1.1.3 Associative law

The "Associative Law" says that it doesn't matter how we group the numbers.

Addition

$$(a + b) + c = a + (b + c)$$

Example 1.4

Show that $(2 + 3) + 2 = (2 + 2) + 3$

$$(2 + 3) + 2 = (2 + 2) + 3$$
$$5 + 2 = 4 + 3$$
$$7 = 7$$
$$\therefore LHS = RHS$$

Multiplication

$$(a \times b) \times c = a \times (b \times c)$$

Example 1.5

$$(2 \times 3) \times 2 = (2 \times 2) \times 3$$
$$6 \times 2 = 4 \times 3$$
$$12 = 12$$
$$\therefore LHS = RHS$$

Activity 1 algebraic expressions and equations Duration: 20min

Write down the general algebraic expression or equation for the questions below. There is more than one solution for some of them, so be inventive! Would these general expressions always hold true? Will they be valid for any number?

1.1 $3(4-7)$

1.2 $13+(14+15)=(13+14)+15$

1.3 $2+2=2\times 2$

1.4 $\frac{1}{\left(\frac{1}{3}\right)}=3$

1.5 $-(-5)=5$

1.6 $21+0=21$

1.7 $13\times 1=13$

1.8 $5\ percent\ of\ 140$

1.9 $\frac{12\times 15}{12+15}$

1.10 $4+5=5+4$

1.11 $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\cdots$

1.2 Terminology used in algebraic expressions

Verbal description	Algebraic expressions
The sum of a number and 2	x + 2 [(sum means addition (+)]
A number that is 5 more than y	y + 5 [more than means addition (+)]
The difference between 2 and a number	x – 2 [difference means subtraction (–)]
A number increased by 7	b + 7 [increase means addition (+)]
The product of 2 and a number	$x \times 2$ OR $2x$ [product means multiplication (×)]
A number that is half the sum of 2 and 3	$c = \frac{(2+3)}{2}$
The quotient if x is divided by 2	$\frac{x}{2}$ [quotient means the answer of a division(÷)]
A number that is decreased by 9	$d - 9$ [decrease means subtraction (–)]

Activity 2	Duration: 20min

2.1 Write these algebraic expressions in mathematical form

a. $x\,4$; $-6 \times y + r$; $m7kh$
b. $z2 + y2$
c. $-\frac{1}{2} \times d \times b \times 6 \div q4$
d. t times 4
e. two thirds of b
f. Add the sum of x and y to the product of p and q
g. Add 21 to the product of a number and 24
h. The difference between two hundred and 133
i. Divide a number by 4 and subtract 9 from the answer

2.2 Interpret the following statement:

a. The relationship between the brother's age(t) and his sister's age is given as $\frac{11}{2} + t$.
b. Themba is x years old and his brother is 4 times his age. Write down an expression to show their total age.

   I. in 3 years' time
   II. in x years' time
   III. 2 years ago.

c. Jobe buys three different domestic animals (goat, sheep and a cow). The goat and a sheep cost $(x + y)$ rands. The cow costs triple the amount of the goat and the sheep. Write an expression that shows the total amount that the three domestic animals cost.

1.3 Adding or subtracting like terms of an algebraic

An expression can have like and unlike terms. Like terms are terms that have the same variable(s), raised to the same power. Constant terms are like terms. Only like terms can be added or subtracted.

Activity 3 addition and subtraction of like terms	**Duration: 10min**
Simplify the following algebraic expressions:	
3.1 $4a^2 + 12a + 62a^2 + 6a$	
3.2 $\frac{1}{2}m^4 + 6m^3 + \frac{1}{4}m^4 + 22$	
3.3 $2z^3 - 7z^3$	
3.4 $31n^2 + 17n - 21n^2 - 6n$	
3.5 $9xyz + 4xy + 6zxy - 15yx$	

1.4 Simplifying algebraic expressions involving exponents and multiple terms

Whenever you simplify algebraic expressions involving exponents and multiple terms, it is necessary to revise the laws of exponents and be mindful of the following key aspects:

- like terms can be added and/or subtracted
- unlike terms cannot be added or subtracted
- addition and subtraction separate terms

Activity 4 algebraic expressions	**Duration: 20min**
Simplify the following algebraic expressions	
4.1 $\frac{2p^3 - px^2}{p}$	
4.2 $\frac{-8x^4 + 4x^3y - 2x^2}{2x}$	
4.3 $x(-2x) - 3x(4x - x^2) + 6(x^3 + 2x^2 + 5)$	
4.4 $\frac{8x^3}{-2} + \frac{2x^3 - x^2}{x} + \frac{2x^4 + 4x^3 - 2x^2}{2x}$	

Unit 2. algebraic equations-Grade 8 & 9

Objectives

At the end of this unit you should be able to:

- set up equations to describe problem situations
- analyse and interpret equations that describe a given situation
- solve equations by inspection
- using additive and multiplicative inverses
- using laws of exponents
- determine the numerical value of an expression by substitution.

2.1 Algebraic equations

Introduction

Solving equation without fractions

There are many ways to solve equations. It is often possible to guess a possible value for the variable and then check if it is a solution using substitution. One method that helps solve more complicated equations uses inverse operations. An inverse operation will undo an operation as seen below:

Addition $\longleftrightarrow$ Subtraction
Multiplication $\longleftrightarrow$ Division

Example 2.2

1. Solve for x given that: $x^2 - 3x - 18$

Step 1: Find two integers whose product is 18 and whose sum is 3

$$(x-6)(x+3)=0$$
$$x=6 \; or \; x=-3$$

Solving equations with fractions

Solving fractional equations is very much like addition and subtraction of fractions, but after the first step, you will get rid of the denominators. “Getting rid” of the denominators is actually changing them all to the value of 1, which of course does not need to be written, leaving us with a denominator-free equation.

Here are some few tips on how to solve Equations with fractions:

What to do.....	Why it works ...
1. Start by choosing the common denominator for the equation.	Terms in equations are often connected by addition or subtraction. Dealing with addition or subtraction of fractions requires a common denominator.
2. Multiply EVERY TERM in the equation by the lowest common denominator.	In an equation, (unlike an expression), you may multiply "every term" on both sides of the equal sign by the same value and not change the equation. You maintain a balanced equation.
3. Reduce each term to form a "denominator free" equation.	All denominators in the equation can be reduced with our lowest common denominator, thus leaving all equation denominators as a value of one.

Example 2.3

Solve for x if: $\frac{2x}{3} + 2 = \frac{11}{5}$

Step 1: Identify common denominator-in this case it is 15

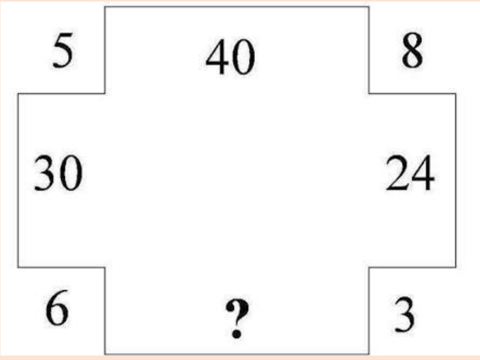
Step 2: multiply each term by lowest common denominator

$$15\left(\frac{2x}{3}\right) + 15(2) = 15\left(\frac{11}{5}\right)$$
$$10x + 30 = 33$$
$$10x = 3$$
$$x = \frac{3}{10}$$

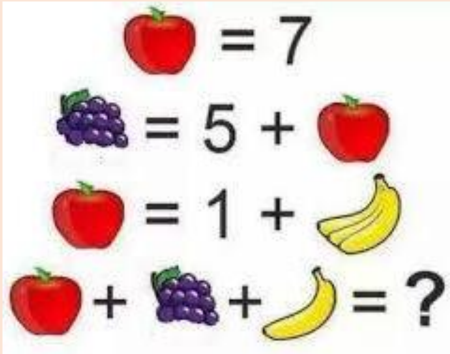
Activity 5: promoting creative thinking **Duration: 20min**

Use your knowledge and skills of the concept of number patterns and equations to answer the quizzes that follow: Show all your workings.

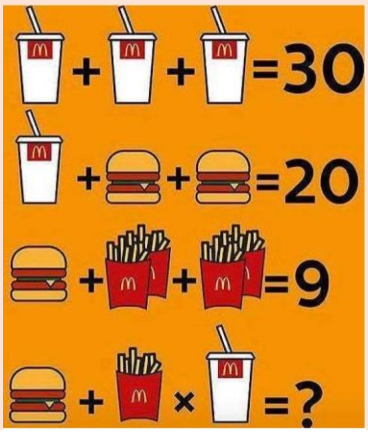
5.1



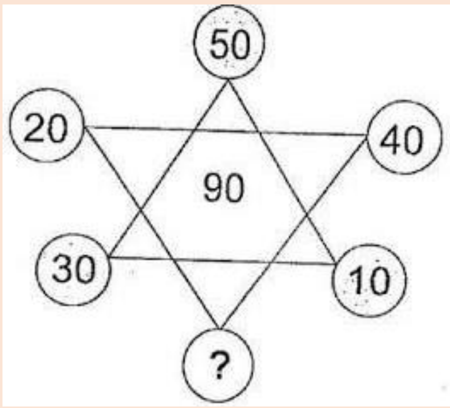
5.2



5.3



5.4



Activity 6 algebraic equations **Duration: 20min**

Solve the following equations:

6.1 $b^2 - 9 = 0$

6.2 $\frac{1}{2}k^2 = 18$

6.3 $\frac{3}{a} - 10 = 0$

6.4 $16^{x-3} = 32$

6.5 $27^{2x+2} = 81^2$

2.2 Solving word problems

Learners must be able to set up simple equation from a real life situation. Hence the mathematical language is important. When solving word problems learners should pay attention to key words such as, sum, difference, product, quotient, double, half, etc. Learners must be able to check or verify the feasibility of their solutions.

Activity 7 word problems		Duration: 30min
7.1	The sum of three consecutive odd numbers is 57. What is the smallest of these numbers?	
7.2	A wise man once said, "500 reduced by twice my age is 334." What is his age?	
7.3	Thandi spent half of her weekly allowance playing Pucman game. To earn more money her parents let her wash the dog for R9.45. What is her weekly allowance if she ended with R19.11?	
7.4	203 students went on a field trip. Four buses were filled and 27 students travelled in cars. How many students were in each bus?	
7.5	Mike's Bikes rents bikes for R100.61 plus R75.00 per hour. Natasha paid R400.61 to rent a bike. For how many hours did she rent the bike?	

Unit 3. Functions and relationships-grade 8 & 9

3.1 Objectives

By the end of the lesson learners should know and be able to determine input values, output values or rules for patterns and relationships using

- tables
- formulae
- equations

3.2 Introduction

What is a Function? The concept of “function” is one that is very important in mathematics. The use of this term is very specific and describes a particular relationship between two quantities: an input quantity and an output quantity. Specifically, a relationship between two quantities can be defined as function if it is the case that “each input value is associated with only one output value”. Why Do We Care About Functions?

Imagine that you are a nurse working the emergency room of a hospital. A very sick person arrives. You know just the medicine needed but you are unsure the exact dose. First, you determine the patient’s mass (80 kg). Then you look at the table below and see the given dosage information:

Person’s mass in kg	Medicine (ml)
80	10
80	100

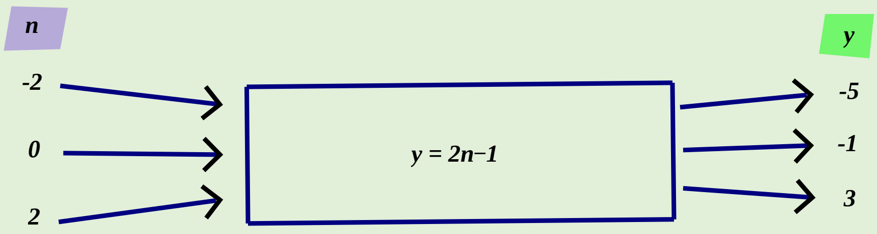
You are immediately confused and very concerned. How much medicine do you give? 10 ml or 100 ml? One amount could be too much and the other not enough. How do you choose the correct amount? What you have here is a situation that does NOT define a function (and would not occur in real life). In this case, for the input value 80 kg, there are two choices for the output value. If you have a function, you will not have to choose between output values for a given input. In the real case of patients and medicine, the dosage charts are based upon functions.

A More Formal Definition of Function is that a function is a rule that relates an input quantity and an output quantity so that a single output value is assigned to each input value. We say that the OUTPUT is a FUNCTION OF the INPUT. Note: Another way to think of this information is that in a functional relationship there are no repeated inputs (with different outputs)

Example 3.1

Use the flow diagram to answer the questions below

a) Which are the input values?
b) Which are the output values?
c) Which one is the rule for this flow diagram?



Answers:

a) $-2,\ 0,\ 2$
b) $-5,\ -1,\ 3$
c) $y = 2n - 1$

Activity 8 functions and relations **Duration: 30min**

8.1 Consider the flow diagram below and use it to complete the table.

$$\text{Inputs} \longrightarrow \boxed{\times\left(-\frac{2}{3}\right)} \longrightarrow \boxed{+5} \longrightarrow \text{Outputs}$$

Input values	1	2	3	4	5	x
Output values						

8.2 In each case:

i. Complete the table.
ii. Write the rule for the nth output.
iii. Show that your rule is correct.

a)

Input value	1	2	3	10	29
Output value	3	5	7		

b)

Input value	2	6	11	60	112
Output value	10	22	37		

c)

Input value	3	7	12			115
Output value	10	50	145	170	6562	

Input value	3	7	12	13	81	115
Output value		50	145	170		

d)

Input value	8	31	66	121	
Output value	81	311	661		1331

Input value	3	7	12	13	81	
Output value	31	50	145			1151

e)

Input value	4	9	49	72	
Output value	10	25	145		496

8.3 Write a formulae that provide the same information as the verbal representations below. Use $\boldsymbol{x}$ for the input number and $\boldsymbol{y}$ for the output number.

a) Multiply the input number by 10, then subtract 3 to get the output number.

b) Multiply the square of the input number by 10, then add 5 times the input number to get the output number.

c) Add 4 to the input number, then subtract the answer from 50 to get the output number.

3.3 Function evaluation

To evaluate a function at a particular value of the input variable, replace each occurrence of the input variable with the given value and compute the result. Use of ( ) around your input value, especially if the input is negative, can help achieve correct results.

Example 3.2

If $f(x) = 5x^2 - 1$, determine $f(2), f(x+1) and\ f\left(-\frac{1}{2}\right)$

$f(2) = 5(2)^2 - 1 = 20 - 1 = 19$
$f(x+1) = 5(x+1)^2 - 1 = 5(x+1)(x+1) - 1 = 5(x^2 + 2x + 1) - 1 = 5x^2 + 10x + 4$
$f\left(-\frac{1}{2}\right) = 5\left(-\frac{1}{2}\right)^2 - 1 = 5\left(\frac{1}{4}\right) - 1 = \frac{5}{4} - 1 = \frac{1}{4}$

Activity 9 substitution **Duration: 15min**

For each of the function below, evaluate $f(3), f\left(-\frac{2}{3}\right)\ and\ f(-2)$

9.1 $f(x) = 2x - 5$

9.2 $f(x) = -4x^2 + 5x + 12$

Diagnostic assessment

Mathematical knowledge gaps

Over the years, learners entering high school struggled to succeed in mathematics. As a result, the South African high schools are faced with the challenge of the mathematical knowledge gap and skills. Mathematical knowledge gap is defined as the lack of smooth transition from primary to high school mathematics due to the shortcoming of the knowledge possessed by the primary school leavers and the knowledge required for high school mathematics curriculum. In order to address this challenge, a comprehensive system of intervention that not only diagnoses gaps, but also prescribes and provides the instructional materials teachers need to address learners' needs, is suggested. Because of the time constrains, we often assume that learners understand the terms and concepts they have learnt in the previous grades.

Teaching for learning

As mathematics teachers we have always focused on assessment of learning, carefully assessing what learners have learned through control tests, class test, mid-year examination, final examination, just to name a few. This reason for assessment is an important component of a mathematics program. However, we should take a broader view of assessment- assessment should be more than merely a test at the end of instruction to see how learners perform … it should be an integral part of instruction that informs and guides us (teachers) as we make instructional decisions. Assessment should not merely be done to learners; rather, it should also be done for learners, to guide and enhance their learning.

Strategies for identifying mathematical knowledge gap

Conceptually, learners will need to have a core foundation and understanding of their multiplication facts, fractions, integers, patterns, space and shape, and how to solve one-step equations.

Such foundational knowledge need to be diagnosed in order to understand how wide the gap is, and who need more support. The purpose of a diagnostic test is to assess the current state of a learner's progress or ability in maths. An in-depth diagnostic assessment focusing on the following topics; is suggested.

PART 1	numerical patterns, functions and relationships, global graphs, and algebraic expressions

PART 2	space and shape (2D and 3D geometry), and measurement (units, perimeter and area)

The test will be divided in two parts each comprising 20 multiple questions (35 minutes) (to be administered during 1 lesson, allowing 10 minutes for administration and collection).

The information about learner performance is vital to the teacher more than any other person, so it is expected that teachers should administer the test, analyse and identify the gap filling activities to assist learner who need more help and provide expanded opportunities to those learners with narrow gaps. This assessment will also provide valuable insight into the foundational skills on which to focus before embarking on grade 8 content.

Analysis

The analysis of the test will be through an excel sheet. The excel sheet will enable you to type in learner's responses and magically do all the analysis and suggest the gap filling activities that learners must attempt. This is to be accompanied by a teacher's information sheet that explains the results analysis and points to resources that can be used (to be shared on a USB).

Gap filling activities

All grade 8 maths teachers will be provided with a USB containing resources that can be used to fill the gaps. As foundational skills are built, learners will be better prepared for learning grade 8 maths. Learners will begin to develop higher-level reasoning skills that will be used throughout the high school mathematics. Starting grade 8 maths with a strong foundational base will be vital to the learner's success. But not all of what learners will need is content based. We feel very strongly that a large part of a learner's math ability is confidence and perseverance. If learners believe they can be successful, they will. This may come after multiple failed attempts, but eventually they will succeed.

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