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Grades 4 to 7
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Resources Handout

Mathematics

CONTENTS

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what I do matters

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APPENDIX 1: CAPS – COMMON FRACTIONS PHASE OVERVIEWS

COMMON FRACTIONS PHASE OVERVIEWS
INTERMEDIATE AND SENIOR PHASE

GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 9
What is different to Grade 3? • Sevenths are new. • Addition of common fractions with the same denominators	**What is different to Grade 4?** • Ninths, tenths, elevenths and twelfths • Learners count in fractions • Subtraction of fractions with the same denominators • Addition and subtraction of mixed numbers • Fractions of whole numbers that result in whole numbers	**What is different to Grade 5?** • Naming, ordering and comparing all common fractions • Tenths and hundredths - this is to lay the basis for decimals and percentages. • Addition and subtraction of common fractions in which one denominator is a multiple of another • Fractions of whole numbers, that result in whole numbers or fractions or both. • Decimal fractions - learners work with decimals to 2 decimal places • Percentages. • The equivalence of common fraction, decimal fractions and percentages	**What is different to Grade 6?** • Compare and order thousandths • Multiplication of common fractions • Percentage of part of a whole • Percentage increase or decrease	**What is different to Grade 7?** • Divide by common fractions • Squares, cubes, square roots and cube roots of common fractions	**What is different to Grade 8?** In Grade 9 learners consolidate number knowledge and calculation techniques for common fractions, developed in Grade 8.
Describing and ordering fractions: • Compare and order common fractions with different denominators (halves; thirds, quarters; fifths; sixths; sevenths; eighths) • Describe and compare common fractions in diagram form	**Describing and ordering fractions:** • Count forwards and backwards in fractions • Compare and order common fractions to at least twelfths	**Describing and ordering fractions:** • Compare and order common fractions, including tenths and hundredths	**Ordering, comparing and simplifying fractions** • Revise the following done in Grade 6: Compare and order common fractions, including specifically tenths and hundredths • Extend to thousandths		

GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 9
Calculations with fractions: • Addition of common fractions with the same denominators • Recognise, describe and use the equivalence of division and fractions	**Calculations with fractions:** • Addition and subtraction of common fractions with the same denominators • Addition and subtraction of mixed numbers • Fractions of whole numbers which result in whole numbers • Recognise, describe and use the equivalence of division and fractions	**Calculations with fractions:** • Addition and subtraction of common fractions in which one denominator is a multiple of another • Addition and subtraction of mixed numbers • Fractions of whole numbers	**Calculations with fractions** • Revise the following done in Grade 6: - addition and subtraction of common fractions, including mixed numbers, limited to fractions with the same denominator or where one denominator is a multiple of another - finding fractions of whole numbers • Extend addition and subtraction to fractions where one denominator is not a multiple of the other • Multiplication of common fractions, including mixed numbers **Calculation techniques** • Convert mixed numbers to common fractions in order to perform calculations with them • Use knowledge of multiples and factors to write fractions in the simplest form before or after calculations • Use knowledge of equivalent fractions to add and subtract common fractions	**Calculations with fractions** • Revise: - addition and subtraction of common fractions, including mixed numbers - finding fractions of whole numbers - multiplication of common fractions, including mixed numbers • Divide whole numbers and common fractions by common fractions • Calculate the squares, cubes, square roots and cube roots of common fractions **Calculation techniques** • Revise: - convert mixed numbers to common fractions in order to perform calculations with them - use knowledge of multiples and factors to write fractions in the simplest form before or after calculations - use knowledge of equivalent fractions to add and subtract common fractions • Use knowledge of reciprocal relationships to divide common fractions	**Calculations with fractions** • All four operations with common fractions and mixed numbers • All four operations, with numbers that involve the squares, cubes, square roots and cube roots of common fractions **Calculation techniques** • Revise: - convert mixed numbers to common fractions in order to perform calculations with them - use knowledge of multiples and factors to write fractions in the simplest form before or after calculations - use knowledge of equivalent fractions to add and subtract common fractions - use knowledge of reciprocal relationships to divide common fractions

GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 9
Solving problems • Solve problems in contexts involving fractions, including grouping and equal sharing	**Solving problems** • Solve problems in contexts involving common fractions, including grouping and sharing	**Solving problems** • Solve problems in contexts involving common fractions, including grouping and sharing **Percentages** • Find percentages of whole numbers	**Solving problems** • Solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing and finding fractions of whole numbers **Percentages** • Revise the following done in Grade 6: - Finding percentages of whole numbers • Calculate the percentage of part of a whole • Calculate percentage increase or decrease of whole numbers • Solve problems in contexts involving percentages	**Solving problems** • Solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing and finding fractions of whole numbers **Percentages** • Revise: - finding percentages of whole numbers - calculating the percentage of part of a whole - calculating percentage increase or decrease • Calculate amounts if given percentage increase or decrease • Solve problems in contexts involving percentages	**Solving problems** • Solve problems in contexts involving common fractions, mixed numbers and percentages
Equivalent forms: • Recognise and use equivalent forms of common fractions (fractions in which one denominator is a multiple of another)	**Equivalent forms**: • Recognise and use equivalent forms of common fractions (fractions in which one denominator is a multiple of another)	**Equivalent forms:** • Recognise and use equivalent forms of common fractions with 1-digit or 2-digit denominators (fractions in which one denominator is a multiple of another) • Recognise equivalence between common fraction and decimal fraction forms of the same number • Recognise equivalence between common fraction, decimal fraction and percentage forms of the same number	**Equivalent forms** Revise the following done in Grade 6: - recognise and use equivalent forms of common fractions with 1-digit or 2-digit denominators (fractions where one denominator is a multiple of the other) - recognise equivalence between common fraction and decimal fraction forms of the same number - recognise equivalence between common fraction, decimal fraction and percentage forms of the same number	**Equivalent forms** Revise equivalent forms between: - common fractions (fractions where one denominator is a multiple of the other) - common fraction and decimal fraction forms of the same number - common fraction, decimal fraction and percentage forms of the same number	**Equivalent forms** Revise equivalent forms between: - common fractions where one denominator is a multiple of another - common fraction and decimal fraction forms of the same number - common fraction, decimal fraction and percentage forms of the same number

APPENDIX 2: CAPS – CLARIFICATION NOTES AND TEACHING GUIDELINES FOR GRADES 4 to 9

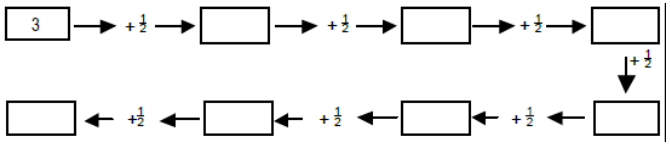
1) Grade 4 Clarification Notes and Teaching Guidelines

GR 4: CONCEPTS AND SKILLS	TERM 2 Page 71 (6 hours)	TERM 3 Page 91 (5 hours)	TERM 4 Page 112 (5 hours)
Solving problems Solve problems in contexts involving fractions, including grouping and equal sharing **Describing and ordering fractions** • Compare and order common fractions with different denominators (halves; thirds, quarters; fifths; sixths; sevenths; eighths) • Describe and compare common fractions in diagram form **Calculations with fractions** • addition of common fractions with the same denominators • recognize, describe and use the equivalence of division and fractions **Equivalent forms** Recognize and use equivalent forms of common fractions with denominators which are multiples of each other	**What is different to Grade 3?** Sevenths are new. There are different ways to understand fractions. This means that learners should develop the concept of fractions in a variety of ways. Problem-solving contexts can help learners to understand many ways of thinking about fractions. A variety of problems should be given to learners. See the types of fractions problems stated *at the end of the Grade notes*. The concept of a fraction should first be developed before learners focus on equivalence and calculating. Learners can also work with apparatus and diagrams. Different diagrams or apparatus develop different ways of thinking about fractions: • Region or area models develop the concept of fractions as part of a whole. If used in particular ways, they can also develop the concept of a fraction as a measure. **Examples** of area models include circles cut into fraction pieces or diagrams of pies, rectangles or other geometric shapes divided into fraction pieces (paper folding), fractions using square or dotty grid paper, geoboards • Length or measurement models can be used to develop the concept of fractions as part of a whole and if used in particular ways also fraction as a measure **Examples** of length models include fraction strips, Cuisenaire rods, number lines • Set models develop the concept of a fraction of a collection of objects and can lay the basis for thinking about a fraction of a number e.g. $\frac{1}{3}$ of 12 **Examples** of set models include counters of any kind in different arrangements	Learners should develop the concept of fractions in a variety of ways, including • a range of problem-solving contexts (see the types of fraction problems stated at the end of the Grade 4 notes). • a range of apparatus and diagrams (see notes Term 1) **Equivalent forms** A focus of Term 2 can be on equivalence (which should be developed through problem-solving and working with diagrams and apparatus). The fractions that learners will be assessed on in Grade 4 were stated in Term 1. Learners are not expected to be able to give equivalent fractions in symbolic (number) form without having diagrams which they can refer or without a problem context in which to make sense of the equivalence. It is recommended that fraction strips or fraction walls are provided when learners are formally assessed on equivalence. **Comparing and ordering fractions:** Learners should also compare and order fractions either with the aid of diagrams (fractions as shapes or number lines) or through problem contexts or using the two together. **Calculations with fractions:** Calculations with fractions are limited to • making fractions through grouping or sharing which is linked with understanding the relationship between division and fractions e.g. If 5 children share sweets equally, they will each get $\frac{1}{5}$ of the sweets • adding fractions with the same denominators Calculations with other aspects of fractions should be developed either through problem contexts or with the use of apparatus or diagrams. Learners should be given problem contexts in which they need to add fraction parts. Learners should also be given either fraction pieces to count e.g. $\frac{3}{8}+\frac{4}{8}$ can be done by counting out and counting on in eighths with apparatus or by colouring in diagrams or by "hopping" in eighths on a number line.	This is revision and consolidation of the concepts developed in Term 3. See Term 3 notes In Term 4 length, capacity and mass can be used as contexts for fractions.

GR 4: CONCEPTS AND SKILLS	TERM 2 Page 71 (6 hours)	TERM 3 Page 91 (5 hours)	TERM 4 Page 112 (5 hours)
	Learners should not only work with one kind of model, because this can limit their understanding of fractions. For example, fractions in diagram forms should include region models (circles and other geometric shapes divided into fraction parts), length models (including number lines) and set models (which show collections of objects). In Term 1 learners should revise and consolidate what they learned about fractions in Grade 3. Learners should solve problems as well as work with apparatus and diagrams involving area, length and set models to ensure that they • understand the relationship between fractions and division i.e. if you share amongst 3 learners you will be making thirds • are able to name fractions. Terminology like "3 over 4" should be avoided as it tends to encourage learners to think about each fraction as two different numbers, rather than $\frac{3}{4}$ being a number which is greater than $\frac{1}{2}$ but less than 1. When naming fraction parts it is useful for learners to rather use the form "3-quarters". Learners should, through work with apparatus, diagrams and solving problems, learn the new fractions that they will deal with in Grade 4.	Measurement is an important context through which to develop and consolidate the notion of fractions. If the suggested sequencing in this document is followed, then learners will have covered length and capacity already. Length and capacity can be used to develop the concepts of fractions, equivalence, and adding with fractions.	

2) Grade 5 Clarification Notes and Teaching Guidelines

Gr 5: CONCEPTS AND SKILLS	TERM 2 Page 160 (5 hours)	TERM 3 Page 176 (5 hours)	TERM 4 Page 199 (5 hours)
Concepts, skills and number range • Describing and ordering fractions • Count forwards and backwards in fractions • Compare and order common • fractions to at least twelfths **Calculations with fractions** • Addition of common fractions with the same denominator • Recognize, describe and use the equivalence of division and fractions **Solving problems** Solve problems in contexts involving common fractions, including grouping and sharing **Equivalent forms:** Recognize and use equivalent forms of common fractions with denominators which are multiples of each other.	What is different to Grade 4? • Ninths, tenths, elevenths and twelfths • Learners count in fractions • Subtraction of fractions with the same denominators • Addition and subtraction of mixed numbers • Fractions of whole numbers that result in whole numbers Most of the new work mentioned above can be developed in Terms 3 and 4. However, learners can begin to count in fractions Learners should develop the concept of fractions in a variety of ways. Problem solving contexts can help learners to understand many ways of thinking about fractions. A variety of problems should be given to learners. See the types of fractions problems stated at the end of the grade notes. Learners can also work with apparatus and diagrams. Different diagrams or apparatus develop different ways of thinking about fractions. • Region or area models develop the concept of fractions as part of a whole. If used in particular ways, they can also develop the concept of fraction as a measure. **Examples of area models** include circles cut into fraction pieces (or diagrams of pies), rectangles or other geometric shapes divided into fraction pieces (paper folding), fractions using square or dotty grid paper, geoboards. • Length or measurement models can be used to develop the concept of fractions as part of a whole and if used in particular ways also fraction as a measure. **Examples of length models** include fraction strips, Cuisenaire rods, number lines. • Set models develop the concept of fraction of a collection of objects (and can lay the basis for thinking about a fraction of a number e.g. $\frac{1}{3}$ of 12) **Examples of set models** include counters of any kind in different arrangements. Learners should not only work with one kind of model, because this can limit their understanding of fractions. For example, fractions in diagram forms should include region model (circles and other geometric shapes divided into fraction parts), length models (including number lines) and set models (which show collections of objects).	Learners should develop the concept of fractions in a variety of ways, including • a range of problem-solving contexts (see the types of fraction problems stated at the end of the Grade 5 notes). • a range of apparatus and diagrams (see notes in Term 1) Learners are not expected to be able to give equivalent fractions in symbolic (number) form without having diagrams to which they can refer or a problem context in which to make sense of the equivalence. It is recommended that fraction strips or fraction walls are provided when learners are formally assessed on equivalence. Once learners are comfortable with equivalence, it is easy for them to compare and order fractions. **Calculations with fractions:** Learners continue to • make fractions through grouping or sharing which is linked with understanding the relationship between division and fractions e.g. If 5 children share sweets equally, they will each get $\frac{1}{5}$ of the sweets • add fractions with the same denominators Calculations as with other aspects of fractions should be developed either through problem contexts or with the use of apparatus or diagrams. Learners should be given problem contexts in which they need to add fraction parts. Learners should also be given either fraction pieces to count e.g. $\frac{3}{8}+\frac{4}{8}$ can be done by counting out and counting on in eighths with apparatus or by colouring in diagrams or by "hopping" in eighths on a number line.	This is further practice of fractions done in Term 3. Refer to those notes. In Term 4 length, capacity and mass can be used as contexts for fractions.
	In Term 2 learners should revise and consolidate what they learned about	Learners are also expected to:	

Gr 5: CONCEPTS AND SKILLS	TERM 2 Page 160 (5 hours)	TERM 3 Page 176 (5 hours)	TERM 4 Page 199 (5 hours)
	fractions in Grade 4. This is described below, but learners can also count in fractions. Counting in fractions can happen • as learners place down fraction pieces • on the number line • or in number chains like the one shown below.  Learners should solve problems as well as work with apparatus and diagrams (area, length and set models) to ensure that they • understand the relationship between fractions and division i.e. if you share equally amongst 3 learners you will be making thirds • are able to name fractions (terminology like "3 over 4" should be avoided as it tends to encourage learners to think about each fraction as two different numbers, rather than $\frac{3}{4}$ being a number which is greater than $\frac{1}{2}$ but less than 1). Learners should, through work with apparatus, diagrams and solving problems, deal with at least the list of fractions required in Grade 4. This should be extended to include the full range of fractions required in Grade 5. The initial focus on fractions should deal with understanding the concept of a fraction. Once learners have consolidated this, they can move on to working with equivalence, then comparing and then calculating with fractions. **Equivalence, comparing and ordering** Equivalence should be approached using apparatus, diagrams or problem contexts. Learners are not expected to be able to give equivalent fractions in symbolic (number) form without having diagrams to which they can refer or a problem context in which to make sense of the equivalence. Once learners are comfortable with equivalence, it is easy for them to compare and order fractions.	• find fractions of whole numbers which result in whole numbers e.g. what is $\frac{1}{4}$ of 24? If learners have worked with drawings of collections of objects, and they know the relationship between division and fractions, this can be done without learning a rule or method. Learners can simply draw 24 objects and then make 4 equal groups • subtract fractions with the same denominators • add and subtract mixed numbers It is not expected that learners know rules for simplifying fractions or for converting between mixed numbers and fraction forms. Learners should know from working with equivalence, when a fraction is equal to or greater than a given fraction **Examples** The examples below are illustrated without contexts but could equally arise in a problem situation. $2\frac{3}{5}+3\frac{4}{5}=5\frac{7}{5}=5+\frac{5}{5}+\frac{2}{5}=6\frac{2}{5}$ Similarly, with subtraction, learners can first subtract the whole numbers, and then use equivalence and compensation to complete the calculation. $6\frac{3}{5}-2\frac{4}{5}=4+\frac{3}{5}-\frac{4}{5}=3+\frac{5}{5}+\frac{3}{5}-\frac{4}{5}=3\frac{4}{5}$ Measurement is an important context through which to develop and consolidate the concept of fractions. If the suggested sequencing in this document is followed, then learners will already have covered length and capacity. Length and capacity can be used to develop the concepts of fractions, equivalence, and adding with fractions.	
	Calculations with fractions:		

Gr 5: CONCEPTS AND SKILLS	TERM 2 Page 160 (5 hours)	TERM 3 Page 176 (5 hours)	TERM 4 Page 199 (5 hours)
	Calculations with fractions in the first term can focus on • making fractions through grouping or sharing which is linked with understanding the relationship between division and fractions e.g. If children share sweets equally, they will each get $\frac{1}{5}$ of the sweets • adding fractions with the same denominators Calculations as with other aspects of fractions should be developed either through problem contexts or with the use of apparatus or diagrams. Learners should be given problem contexts in which they need to add fraction parts. Learners should also be given either fraction pieces to count e.g. $\frac{3}{8}+\frac{4}{8}$ can be done by counting out and counting on in eighths with apparatus or by colouring in diagrams or by "hopping" in eighths on a number line.		

3) Grade 6 Clarification Notes and Teaching Guidelines

Gr 6: CONCEPTS AND SKILLS	TERM 1 Page 226 (10 hours)	TERM 4 Page 280 (5 hours)
Describing and ordering fractions: Compare and order common fractions, including specifically tenths and hundredths\ **Calculations with fractions:** • Addition and subtraction of common fractions in which one denominator is a multiple of another • Addition and subtraction of mixed numbers • Fractions of whole numbers **Solving problems** Solve problems in contexts involving common fractions, including grouping and sharing **Percentages** Find percentages of whole numbers **Equivalent forms:** **Learners should recognize** • equivalent forms of common fractions with 1-digit or 2-digit denominators (denominators which are multiples of each other)	**What is different to Grade 5?** • In Grade 6 learners name, order and compare all common fractions • There is a special focus on tenths and hundredths in Grade 6. This is to lay the basis for decimals and percentages. • In Grade 5 learners only added and subtracted common fractions with the same denominator. In Grade 6 they use their knowledge of equivalence and add and subtract common fractions in which one denominator is a multiple of another • When learners calculate fractions of whole numbers, the answers may contain whole numbers or fractions or both. • Decimal fractions are introduced. Learners work with decimals to 2 decimal places • Percentages are introduced. • Learners work with equivalence between - common fraction and decimal fraction forms of the same number - common fraction and percentage forms of the same number - decimal fraction and percentage forms of the same number In Term 1 learners focus on common fractions, which will then be consolidated in Term 4. Learners start by focusing on the meaning of a fraction. Learners should develop the concept of fractions in a variety of ways. Problem-solving contexts can help learners to understand many ways of thinking about fractions. A variety of problems should be given to learners. (See the types of fraction problems stated at the end of the grade notes). Learners can also work with apparatus and diagrams. Different diagrams or apparatus develop different ways of thinking about fractions: • Region or area models develop the concept of fractions as part of a whole. They can also develop the concept of a fraction as a measure. **Examples** of area models include circles cut into fraction pieces or diagrams of pies, rectangles or other geometric shapes divided into fraction pieces by paper folding, fractions using square or dotty grid paper, geoboards • Length or measurement models can be used to develop the concept of fractions as part of a whole and if used in particular ways also of a fraction as a measure **Example:** length models include fraction strips, Cuisenaire rods, number lines. • Set models develop the concept of a fraction of a collection of objects and can lay the basis for thinking about a fraction of a number e.g. $\frac{1}{3}$ of 12. **Examples** of set models include counters of any kind in different arrangements Learners should not only work with one kind of model, because this can limit their understanding of fractions. For example, fractions in diagram forms should include region models e.g. circles and other geometric shapes divided into fraction parts, length models including number lines and set models which show collections of objects. Special attention should be paid to tenths and hundredths as learners will need this background when they work with decimal fractions up to 2 decimal places. Learners have been naming fractions since Grade 2. Extending the range of common fractions should not cause difficulties. Time should rather be spent on equivalence, comparing fractions and doing calculations with fractions and calculating. Once learners are comfortable with equivalence, it is easy for them to compare and order fractions.	This is revision and consolidation of the concepts developed in Term 2. See Term 1 notes. However, since decimals and percentages have both been done, it is useful to practise equivalence between the common fraction, decimal fractions and percentage forms of the same number in Term 4.

Gr 6: CONCEPTS AND SKILLS	TERM 1 Page 226 (10 hours)	TERM 4 Page 280 (5 hours)
• equivalence between common fraction and decimal fraction forms of the same number • equivalence between common fraction, decimal fraction and percentage forms of the same number	**Calculations with fractions:** Learners continue to • determine fractions through grouping or sharing which is linked to understanding the relationship between division and fractions e.g. If 5 children share sweets equally, they will each get $\frac{1}{5}$ of the sweets • add fractions with the same denominators • subtract fractions with the same denominators • add and subtract mixed numbers. It is not expected that learners know rules for simplifying fractions or for converting between mixed numbers and fraction forms. Learners should know when a fraction is equal to or greater than 1. **Examples** The examples below are illustrated without contexts but could equally arise in a problem situation. $2\frac{3}{5}+3\frac{4}{5}=5\frac{7}{5}=5+\frac{5}{5}+\frac{2}{5}=6\frac{2}{5}$ Similarly, to do subtraction, learners can first subtract the whole numbers and then use equivalence and compensation to complete the calculation. $6\frac{3}{5}-2\frac{4}{5}=4+\frac{3}{5}-\frac{4}{5}=3+\frac{5}{5}+\frac{3}{5}-\frac{4}{5}=3\frac{4}{5}$ Learners also find fractions of whole numbers. In Grade 6 learners should do examples in which the answer can be a whole number, a fraction or a mixed number e.g. What is $\frac{1}{4}$ of 18? If learners have worked with pictures of collections of objects, and they know the relationship between division and fractions, this can be done without learning a rule or method. Learners can simply draw 18 objects and then create 4 equal groups. An extension of this question would be to find $\frac{3}{4}$ of 18. Here learners first calculate $\frac{1}{4}$ of 18 and use multiplication to find the answer. Measurement is an important context through which to develop and consolidate the fraction concept. Length, mass and capacity can be used to develop the concepts of fractions, equivalence, and adding with fractions, since learners have worked with these since Grade 1.	

4) Grade 7, 8 and 9 Clarification Notes and Teaching Guidelines

GRADE 7 Page 49 (9 hours)	GRADE 8 Page 100 (7 hours)	GRADE 9 Page 122 (4,5 hours)
What is different to Grade 6? • Compare and order thousandths • Multiplication of common fractions • Percentage of part of a whole • Percentage increase or decrease In Grade 7 learners also consolidate number knowledge and calculation techniques for common fractions, developed in the Intermediate Phase. **Calculations with fractions** • Learners should do context free calculations and solve problems in contexts. • It is not expected that learners know rules for simplifying fractions or for converting between mixed numbers and fraction forms. Learners should know from working with equivalence, when a fraction is equal to or greater than 1. • LCMs have to be found when adding and subtracting fractions of different denominators. Here learners use knowledge of common multiples to find the LCM i.e. what number can both denominators be divided into. • To simplify fractions, learners use knowledge of common factors i.e. what can divide equally into the numerator and denominator of a fraction. Emphasize that when simplifying, the fractions must remain equivalent. **Example**: $\frac{3}{4}\times\frac{2}{5}=\frac{4}{20}=\frac{3}{10}$ **OR** $\frac{3}{4}\times\frac{2}{5}=\frac{3}{10}$	**What is different to Grade 7?** • Divide by common fractions • Squares, cubes, square roots and cube roots of common fractions In Grade 8 learners consolidate number knowledge and calculation techniques for common fractions, developed in Grade 7. **Calculations using fractions** • Learners should continue to do context free calculations and solve problems in contexts. - By Grade 8 learners should be comfortable converting mixed numbers to common fractions for calculations. **Example:** $5\frac{1}{2}=\frac{11}{2}$; $6\frac{1}{3}=\frac{19}{3}$ - To simplify fractions, learners use knowledge of common factors i.e. what can divide equally into the numerator and denominator of a fraction. Emphasize that when simplifying, the fractions must remain equivalent. **Addition and subtraction** • LCMs have to be found when adding and subtracting fractions with different denominators. Here learners use knowledge of common multiples to find the LCM i.e. what number can both denominators be divided into. **Multiplication** • For multiplication of fractions, learners should be encouraged to simplify fractions by dividing numerators and denominators by common factors. • Learners should note the difference between adding or subtracting fractions, and multiplying fractions **Examples**: $\frac{3}{4}+\frac{2}{5}=\frac{15}{20}+\frac{8}{20}=\frac{23}{20}=1\frac{3}{20}$ (using LCM and equivalent fractions) $\frac{3}{4}\times\frac{2}{5}=\frac{3}{10}$ (divide 2 and 4 by common factor 2) • Learners should recognize that finding a 'fraction ***of*** a whole number' or 'finding a fraction ***of*** a fraction' means multiplying the fraction and the whole number or the fraction with the fraction.	**What is different to Grade 8?** In Grade 9 learners consolidate number knowledge and calculation techniques for common fractions, developed in Grade 8. In Grade 9, learners work with common fractions mostly as coefficients in algebraic expressions and equations. They are expected to be competent in performing multiple operations using common fractions and mixed numbers, applying properties of rational numbers appropriately. They are also expected to recognize and use equivalent forms for common fractions appropriately in calculations and when simplifying algebraic fractions.
• Learners should recognize that finding a 'fraction ***of*** a	• When learners find fractions of whole numbers, the examples can be chosen to result	

GRADE 7 Page 49 (9 hours)	GRADE 8 Page 100 (7 hours)	GRADE 9 Page 122 (4,5 hours)
whole number' or 'finding a fraction ***of*** a fraction' means multiplying the fraction and the whole number or the fraction with the fraction. • When learners find fractions of whole numbers, the examples can be chosen to result either in whole numbers or fractions or both. • Learners should also use the convention of writing the whole number as a fraction over 1 when multiplying. **Examples** a) Calculate $\frac{4}{5}$ of 20 $\frac{4}{5}$ of $20 = \frac{4}{5} \times \frac{20}{1} = \frac{4}{1} \times \frac{4}{1} = 16$ OR $\frac{4}{5}$ of $20 = \frac{4}{5} \times \frac{20}{1} = \frac{80}{5} = 16$ b) Calculate $\frac{2}{3}$ of $\frac{5}{6}$ $\frac{2}{3}$ of $\frac{5}{6} = \frac{2}{3} \times \frac{5}{6} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9}$ OR $\frac{2}{3}$ of $\frac{5}{6} = \frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$	either in whole numbers or fractions or both. • Learners should also use the convention of writing the whole number as a fraction over when multiplying. **Examples** Find $\frac{4}{5}$ of $20 = \frac{4}{5} \times \frac{20}{1} = 16$ Find $\frac{2}{3}$ of $\frac{5}{6} = \frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$ **Division** • The technique of 'invert and multiply' applies to division in general and not just to division by fractions. Hence, a useful way of making learners comfortable with division by fractions is to start with examples of division by whole numbers. • Learners have to understand that dividing by a number is the same as multiplying by the reciprocal of the number i.e. the reciprocal of *n* is $\frac{1}{n}$ **Examples**: a) $10 \div 5$ is the same as $10 \times \frac{1}{5} = 2$ (multiply by the reciprocal of 5) b) $10 \div \frac{1}{5} = 10 \times 5 = 50$ (multiply by the reciprocal of $\frac{1}{5}$) This can also be explained by using diagram models for fractions and asking, how many times does $\frac{1}{5}$ fit into 10? We know that 5 fifths fit into 1 whole, so (5×10) fifths will fit into 10 wholes. Hence, $10 \div \frac{1}{5} = 50$ c) $20 \div 4$ is the same as $20 \times \frac{1}{4} = 5$ (multiply by the reciprocal of 4) d) $20 \div \frac{1}{4} = 20 \times 4 = 80$ (multiply by the reciprocal of $\frac{1}{4}$) This can also be explained by using diagram models for fractions and asking, how many times does $\frac{1}{4}$ fit into 20? We know that 4 quarters fit into 1 whole, so (4×20) quarters will fit into 20 wholes. Hence, $20 \div \frac{1}{4} = 80$ e) Once learners have done a few of the above examples, they can use the technique of multiplying by the reciprocal to divide fractions by fractions: $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$ (multiply by the reciprocal of $\frac{1}{2}$)	
	Squares, cubes, square roots and cube roots	

GRADE 7 Page 49 (9 hours)	GRADE 8 Page 100 (7 hours)	GRADE 9 Page 122 (4,5 hours)
Calculation using percentages • Learners should do context free calculations and solve problems in contexts. • When doing calculations using percentages, learners have to use the equivalent common fraction form, which is a fraction with denominator 100. Learners should become familiar with the equivalent fraction and decimal forms of common percentages like a) 25% or $\frac{1}{4}$ or 0,25; b) 50% or $\frac{1}{2}$ or 0,5; c) 60% or $\frac{3}{5}$ or 0,6. • To calculate percentage of part of a whole, or percentage increase or decrease, learners have to learn the strategy of multiplying by $\frac{100}{1}\%$. It is useful for learners to learn to use calculators for some of these calculations where the fractions are not easily simplified. • When using calculators, learners can also use the equivalent decimal fraction form for percentages to do the calculations. **Examples:** a) Calculate 60% of R105 60% of R105 = $\frac{3}{5} \times$ R105 = R63 b) What percentage is 40c of R3,20? Percentage = $\frac{40}{320} \times 100\%$ = 12,5% c) Calculate the percentage increase if the price of a bus ticket of R60 is increased to R84. Increase = R84 – R60 = R24. So, percentage increase = $\frac{24}{60} \times \frac{100}{1}\%$ = 40%	Knowing the rules of operations for calculating squares, cubes, square roots and cube roots of common fractions is important **Examples** a) $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$ b) $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$ Once learners are comfortable doing all the operations with fractions, calculations do not have to be restricted to positive fractions. **Calculation using percentages** • Learners should continue to do context free calculations and solve problems in contexts. • When doing calculations using percentages, learners have to use the equivalent common fraction form, which is a fraction with denominator 100. • Learners should become familiar with the equivalent fraction and decimal forms of common percentages e.g. 25% is equivalent to $\frac{1}{4}$ or 0,25; 50% is equivalent to $\frac{1}{2}$ or 0,5; 60% is equivalent to $\frac{3}{5}$ or 0,6. • To calculate percentage of part of a whole, or percentage increase or decrease, learners have to learn the strategy of multiplying by $\frac{1}{100}\%$. It is useful for learners to learn to use calculators for some of these calculations where the fractions are not easily simplified. • When using calculators, learners can use the equivalent decimal fraction form for percentages to do the calculations. **Examples:** a) Calculate of 60% of R105; Amount = $\frac{60}{100} \times$ R105 = R63 b) What percentage is 40c of R3,20? Percentage = $\frac{40}{320} \times \frac{100}{1}\% = \frac{100}{8}\% = 12{,}5\%$ c) Calculate the percentage increase if the price of a bus ticket of R60 is increased to R84. Amount increased = R24. Therefore, percentage increase $= \frac{24}{60} \times \frac{100}{1}\% = 40\%$ d) Calculate the percentage decrease if the price of petrol goes down from 20 cents a litre to 18 cents a litre. Amount decreased = 2 cents. Therefore, percentage decrease $\frac{2}{20} \times \frac{100}{1}\% = 10\%$	
d) Calculate the percentage decrease if the price of	e) Calculate how much a car will cost if its original price of R150 000 is reduced by	

GRADE 7 Page 49 (9 hours)	GRADE 8 Page 100 (7 hours)	GRADE 9 Page 122 (4,5 hours)
petrol goes down from 20 cents a litre to 18 cents a litre. Decrease = 20 cents – 18 cents = 2 cents. So, percentage decrease $= \frac{2}{10} \times 100\% = 10\%$	15% Calculation involves finding 15% of R150 000 and then subtracting that amount from the original price. i.e. $\frac{15}{100} \times \frac{\text{R150 000}}{1} = \text{R22 500}$ Hence new price of car = R150 000 – R22 500 = R127 500 Or 100% 15% = 85% or 0,85, so new price of car = 0,85 × R150 000 R127 500	

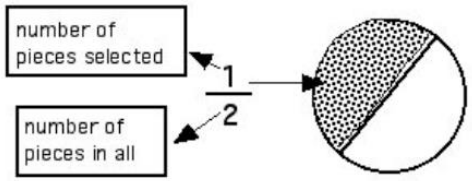
APPENDIX 3: FIVE MEANINGS OF FRACTIONS

This article is found at http://ion.uwinnipeg.ca/~jameis/MY%20course/MY%20readings/5fracmean.pdf

There are five meanings of fraction that are relevant to middle year mathematics. The first three meanings should be developed in the early years (although the measure meaning may have only been developed by some teachers). The fourth and fifth meanings need to be developed in the middle years.

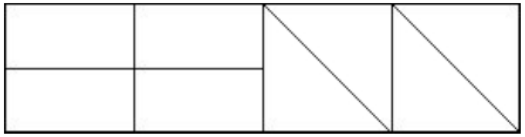
MEANING 1. <u>The cut meaning (also known as the part of a whole meaning)</u> – to be developed in the Foundation Phase and Intermediate Phase

When a 5-year old child says, "I ate half of the cookie." he/she is expressing a *part-whole relationship*. The child uses 'half' not in the sense of a number but in the sense of an actual or imagined action that involves cutting a whole physical object in the middle. The imagined or actual action of cutting a whole object into '*n*' equivalent/equal parts underlies the cut meaning of fraction. We represent each part symbolically by the fraction notation $\frac{1}{n}$. The circle diagram here indicates this.



'Equivalent/equal' means equivalent/equal according to length, area, or volume. In the case above, the two pieces of the circle are equal in area and they happen to look the same. That does not need to be the case.

Consider a granola bar cut in the way shown in the diagram. The 8 pieces do not all look the same. Yet each piece is $\frac{1}{8}$ of the granola bar because the pieces have the same area.



In summary, the cut meaning of fraction involves cutting a naturally existing whole into equal parts according to measurable qualities such as length, area, volume, mass, etc.

- An example for *length* could be a string cut into 4 parts of equal length. Each part is $\frac{1}{4}$ of the length of the whole string.
- An example for *area* is the granola bar or circle example above.
- An example for *volume* could be a loaf of bread cut into 8 parts having the same volume (a difficult thing to actually do). Each part is $\frac{1}{8}$ of the volume of the whole loaf.

MEANING 2. <u>The part of a group or set meaning</u> – to be developed in the Foundation Phase and Intermediate Phase

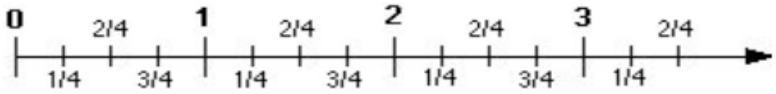
The *part of a group* meaning does not involve cutting a whole into equal parts. Nor does it involve a natural whole. The part of a group meaning involves selecting objects from a group on some basis. A group is not a natural whole as is a pie (for example). Unlike the cut meaning, the part of a group meaning does not require that the objects be of the same size or type.

For example, suppose there are 23 books of varying size and content on a shelf where 14 of them are novels. We can represent this situation by the fraction $\frac{14}{23}$. In this case, we mean by $\frac{14}{23}$ that 14 out of the 23 books are novels. This meaning is significantly different from the 'part of the whole' meaning. The part of a group/set meaning involves mentally placing discrete things into categories (e. g. red, prime), a different task to cutting up things according to length, area, or volume, etc.

MEANING 3. <u>The name for a point (or measure) meaning</u> – to be developed in the Foundation Phase and Intermediate Phase

The *name for a point meaning* involves associating marks on measuring devices such as rulers with fraction names. The measure meaning underlies this as the marks are obtained by cutting a section of a line into equal parts.

Notice in the number line here that there are a lot of $\frac{1}{4}$ lengths (for example). Yet each is different because they each name a different point on the number line (e. g. $\frac{1}{4}, 1\frac{1}{4}, 2\frac{1}{4}$, etc.).



The *name for a point meaning* is the most abstract of the first three meanings of fraction, and, overall, the most useful for purposes of teaching computational skills.

A number line that contains fraction-named points can serve as a ruler for measuring length using fractional amounts of units of length. For this reason, some refer to the name for a point meaning as the measure meaning of fraction.

MEANING 4. <u>Ratio meaning</u> – to be developed in the Senior Phase

A fraction can be used to name a ratio. For example, $\frac{3}{4}$ can mean a ratio of 3 to 4. This has nothing to do with wholes, groups, or name for a point. It involves making a comparison between two quantities.

Ratio is a statement about a numerical relationship (specifically, a comparison) between two quantities that may or may not involve different kinds of stuff.

- Suppose the ratio between flour and butter in a recipe is 5 cups flour to 3 tablespoons butter $\left(5 : 3 \text{ or } \frac{5}{3}\right)$. In this situation, the relationship is between the same kind of stuff, volume, but it does involve different units.
- Suppose that in an eco-system, the optimal ratio between deer and forage is 1 deer to 2 tons of forage per square mile $\left(1 : 2 \text{ or } \frac{1}{2}\right)$. This ratio involves different kinds of stuff, a count of discrete entities (deer) and a measurement of weight and area.

Ratio may be explicit as in the examples above or it may be implicit. If I have two-thirds $\left(\frac{2}{3}\right)$ as many notebooks as Harry, this implies a ratio of 2 to 3 (I have 2 notebooks for each 3 that Harry has).

We can name ratios using a variety of notations $\left(\text{e.g. } 5 \text{ to } 3 \text{ or } 5 : 3 \text{ or } \frac{5}{3}\right)$. Fraction notation has an advantage when working with ratios. For example, we can equate two ratios when we are solving problems about similar triangles $\left(\text{e.g. } \frac{x}{5} = \frac{7}{10}\right)$. Fraction notation leads to relatively simple methods for solving such equations.

MEANING 5. <u>Indicated division meaning</u> – to be developed in the Senior Phase

The *indicated division meaning* has nothing to do with wholes, groups, or measurement. It refers to the fact that the fraction bar, '/', can be a way of saying to divide. One application of this meaning is converting fraction notation to decimal notation, in which we divide the numerator of a fraction by its denominator. For example, $\frac{1}{2}$ is $1 \div 2$ or 0,5 and $\frac{3}{4}$ is $3 \div 4$ or 0,75.

Some of the confusion about 'of' and '×' when working with fractions can be attributed to a lack of understanding that the symbol, '/' can be used to indicate 'divide'. In other words, there is another notational agreement for divide besides the symbol '÷'.

APPENDIX 4: TYPES OF FRACTION PROBLEMS LISTED IN THE INTERMEDIATE PHASE CAPS

DIFFERENT TYPES OF FRACTIONS	EXAMPLES OF PROBLEMS		
	GRADE 4 (page 121)	**GRADE 5 (page 211)**	**GRADE 6 (page 292)**
TYPE 1: Part of a whole (the whole is a single object)	Susan eats two eighths of a chocolate bar. What fraction of the chocolate bar is left? Show your answer in a drawing.	Susan eats $\frac{1}{3}$ of a chocolate cake. Another $\frac{1}{4}$ is given away. How much cake is left over?	Susan eats one half of a chocolate bar. The remainder is equally divided between two friends. How much does each one get? Show your answer in a drawing.
TYPE 2: Parts of a whole (the whole is a collection of objects)	Five friends share 21 chocolates equally. How many chocolates does each person get?	A wall has 124 panels. A painter paints $\frac{1}{3}$ of these panels. How many panels has he painted? How many panels must still be painted? OR Sue uses $\frac{2}{3}$ of an apple to make a cake. If she has 30 apples, how many cakes can she make?	During the holidays Avril spends $\frac{1}{3}$ of his day watching TV and $\frac{1}{4}$ of his day sleeping. How many hours of his day are left?
TYPE 3: Relationship	Barry earns a third of what his father earns per hour. If his father earns R267 per hour, how much does Barry earn per hour?	The daughter earns a quarter of what her father earns per hour. If her father earners R267 per hour, how much does the daughter earn?	The son earns $\frac{20}{100}$ of what his father earners per month. If his father earns R18 000 per month, how much does the son earn?
TYPE 4: Ratio	$\frac{2}{5}$ of a cup of milk is needed to make one batch of biscuits. How many cups of milk are needed to make 5 batches of these biscuits?	The recipe says that for every 2 cups of sugar, $\frac{1}{4}$ cup of butter is needed. If 50 cups of sugar are used, how many cups of butter are needed?	$\frac{2}{5}$ cup of milk is needed to make 40 biscuits. How many cups are needed for 2 000 biscuits? Or Is 10 litres of milk enough to make 2 000 of these biscuits?
TYPE 5: Comparator	Which is longer? $\frac{2}{3}$ of a metre or $\frac{1}{4}$ of a metre?	Which is longer? $\frac{6}{9}$ of a metre or $\frac{2}{3}$ of a metre of material?	Which is longer? $\frac{6}{100}$ of a metre or $\frac{7}{10}$ of a metre of material?
TYPE 6: Unit of measurement	I need $1\frac{2}{5}$ of a metre of material to make a shirt and I have $\frac{4}{5}$ of a metre. How much material do I still need to buy?	How many $\frac{1}{3}$ of a metre are there in $5\frac{2}{3}$ metre?	Nomfundo needs $2\frac{2}{10}$ metres of rope to make a basket. How many baskets can she make with $28\frac{1}{5}$ metres of rope?
TYPE 7: Number	Name two numbers between $4\frac{1}{2}$ and 5.	Give a number that is greater than $3\frac{2}{3}$, but less than $3\frac{11}{12}$	Indicates the positions of the numbers $0, 1, \frac{8}{10}, \frac{2}{5}, 1\frac{40}{100}$ on a number line.
TYPE 8: Fractional parts put together to make a whole (iterative)	After a game, 55 athletes get $\frac{1}{2}$ of an orange each. How many oranges are needed for the 55 athletes?	35 children get cool drink. If each child gets $\frac{2}{11}$ of a bottle of cool drink, how many bottles are needed to serve all the children?	On a sports day, 500 children get $\frac{20}{100}$ of a bottle of cool drink and $\frac{4}{10}$ of a bar of chocolate. How many bottles of cool drink and bars of chocolate are needed to serve all the children?
TYPE 9: Operator			Calculate $\frac{2}{3} \times 336$

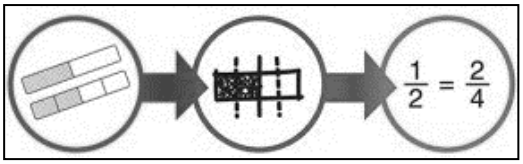
APPENDIX 5: CONCRETE, SEMI-CONCRETE AND ABSTRACT METHODS OF TEACHING

The following article is adapted from the following two websites

- https://makingeducationfun.wordpress.com/2012/04/29/concrete-representational-abstract-cra/
- http://www.specialconnections.ku.edu/?q=instruction/mathematics/teacher_tools/concrete_to_representational_to_abstract_instruction

CONCRETE → SEMI-CONCRETE → ABSTRACT

In order for learners to fully develop an understanding of the maths concepts or skills they are learning, the teacher should move through a **concrete** to **semi concrete** (also called representational) to **abstract sequence of instruction**



Concrete **Semi Concrete** **Abstract**

STEPS TO GO THROUGH WHEN TEACHING FRACTION CONCEPTS

1. **The Concrete Phase**
   First use appropriate concrete objects to teach particular math concepts/skills. Learners can see and feel the attributes of the objects they are using.
   AREA MODELS: plastic circles cut into pie pieces; rectangular regions; geoboard; paper folding
   LENGTH MODELS: Cuisenaire rods which have pieces in lengths from 1 to 10 with each length a different colour; folded paper strips
   SET MODELS: Counters; objects such as beans, unifix cubes, sucker sticks.

2. **The Semi-Concrete Phase**
   After the learners have mastered a concepts at the concrete level, then teach appropriate **drawing techniques** where learners problem solve by drawing pictures of the concrete objects they previously used. By repeating the activities, they have already done using concrete material, the learners abstract understanding of the concept/skill is evolving.
   AREA MODELS: drawings on squared paper or dot paper
   LENGTH MODELS: number lines, Cuisenaire rods drawn on squared paper
   SET MODELS: drawings of different objects

3. **The Abstract Phase**
   Once the learners have understood the concepts at the semi-concrete phase, move on to the abstract level of understanding for a particular math concept/skill i.e. move onto **calculations** only. After the learners have mastered the skill at the abstract level of understanding, make sure the learners maintain their skill level by providing periodic practice opportunities for the math skills

BENEFITS OF THIS APPROACH TO THE TEACHING OF MATHS CONCEPTS

- It provides learners with a structured way to learn math concepts
- Learners are able to build a better connection when moving through the levels of understanding from concrete to abstract
- It makes learning accessible to all learners (including those with math learning disabilities)
- It helps learners learn concepts before learning rules
- It can be used in small groups or entire class
- Research has proven that this method is effective
- It can be used across grade levels, from the Foundation Phase to the FET Band

NEGATIVES OF THIS APPROACH TO THE TEACHING OF MATHS CONCEPTS

- Not commonly used in the Senior Phase (though it should be).

Further resources which you might want to read

1) "Why Is Doing Arithmetic With Fractions So Difficult?" at https://www.psychologytoday.com/intl/blog/talking-apes/201709/why-is-doing-arithmetic-fractions-so-difficult

2) "A componential view of children's difficulties in learning fractions" at https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3794363/

3) "Fractions: Where It All Goes Wrong" at https://www.scientificamerican.com/article/fractions-where-it-all-goes-wrong/?print=true