education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Grades 4 to 7
Just-in-Time Training Workshop
2019: No.1

Participant's Handout

Mathematics

Endorsed by:

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what I do matters

WORKSHOP PROGRAM – GRADES 4 to 7

Session 1: 2 hours	**DEVELOPING FRACTION CONCEPTS** **1.1 Engaging with the Intermediate and Senior Phase CAPS documents** Use Appendix 1 to list the Grades where each fraction concept is introduced and taught. **1.2 Three different models used to develop the concept of fractions** Use Appendix 2 to describe the three models and to give examples of these models. **1.3 Five meanings of fractions** Use Appendix 3 to investigate the five meanings of fractions. **1.4 Types of fraction problems** Use Appendix 4 to write a short description of the Grade 4, Grade 5 and Grade 6 problems given for the Type of fraction allocated to you. Make up your own Grade 7 problem, similar to the Grade 4, 5 and 6 problems you have been allocated. Share them with the class.
Session 2: 1 hour 30 minutes	**USING CONCRETE APPARATUS TO TEACH FRACTIONS** **2.1 What is meant by concrete, semi-concrete and abstract methods of teaching?** Write down what you understand by these terms. These terms are then explained using Appendix 5 **2.2 Using Concrete and Semi-Concrete Material to teach the multiplication and division of fractions** The following topics are covered a) Multiplication of fractions b) Division of fractions c) Using abstract reasoning to explain why we change division to multiplication and invert the second number
Session 3: 1 hour 30 minutes	**USING MAGIC SQUARES TO PRACTICE FRACTION CALCULATIONS** **3.1 What is a Magic Square?** What is a Magic Square and how do we use it? **3.2 How to design a 3 × 3 Magic Square** Learn to design your own 3 × 3 Magic Square **3.3 How to design a 4 × 4 Magic Square.** Learn how to design your own 4 × 4 Magic Square **3.4 Converting your Magic Square into an Activity** Select a sequence of 9 fractions, create your own Magic Square and then turn it into an addition and subtraction activity. Discuss the usefulness of Magic Squares.

SESSION 1: DEVELOPING FRACTION CONCEPTS

1.1. Engaging with the Intermediate and Senior Phase CAPS documents

Work with your group
Use APPENDIX 1 (on pages 1 to 3 in the Resources Handout) to answer the following questions.

1) Both the Intermediate and Senior Phase CAPS documents list, for each grade, the concepts that are different to the previous grade.
   What concepts are new to the learners in

   a) Grade 4?

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   b) Grade 5?

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   c) Grade 6?

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   d) Grade 7?

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e) Grade 8?

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f) Grade 9?

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2) In which Grades do the learners ‘Compare and Order Fractions’? ……………………………

3) Study the references to Addition and Subtraction in the Phase Overview. In which Grades do the learners

a) Start adding fractions with the same denominators? ……………………………………..

b) Start subtracting fractions with the same denominator? ………………………………………

c) Start adding and subtracting fractions where one denominator is a multiple of another? …………..

d) Start adding and subtracting fractions where one denominator is NOT a multiple of the other? ……

e) Start working with the addition and subtraction of mixed numbers? ……………………………

4) Study the references to the Equivalence of Division and Fractions

a) What do we mean by the equivalence of Division and Fractions? ……………………………

b) In which Grades do the learners ‘recognise, describe and use the equivalence of division and fractions’? ……………………………………………………………………………………...

5) Study the references to Multiplication and Division in the Phase Overview. In which Grades do the learners

a) Find the fractions of whole numbers which result in whole number answers? ………………….

b) Find the fractions of whole numbers which do not necessarily result in whole number answers? …………………………………………..

c) Multiply common fractions and mixed numbers? ……………………………………………

d) Divide fractions? ……………………………………………………………………………….

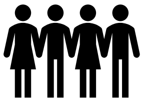
6) In which Grades do the learners calculate the squares, cubes, square roots and cube roots of common fractions? ………………………………………………………………………………………...

7) In which Grades do the learners solve problems in contexts involving fractions? …………………...

8) Study the references to Percentages. In which Grade do the learners
   a) Start working with percentages? ………………………………………………………………
   b) Find the percentages of whole numbers? …………………………………………………...
   c) Calculate the percentage of parts of a whole? …………………………………………………...
   d) Calculate percentage increase or decrease? …………………………………………………..
   e) Calculate amounts if given percentage increase or decrease? …………………………………….
   f) Solve problems in contexts involving percentages? …………………………………….

9) Study the references to Equivalent Forms in the Phase Overview. In which Grades do the learners
   a) Recognise and use equivalent forms of common fractions where one denominator is a multiple of another? ………………………………………………………………………………………...
   b) Recognise the equivalence between common fractions, decimal fractions and percentage forms of the same number? ……………………………………………………………………………..

1.2. <u>Three different models used to develop the concept of fractions</u>



Work with your group
Use APPENDIX 2 (on pages 4 to 14 in the Resources Handout) to answer the following questions.

1) The Clarification Notes for Grade 4, for Grade 5 and for Grade 6 CAPS state that learners should work with apparatus and diagrams to develop different ways of thinking about fractions. These different models are
   - Region or Area Models
   - Length or Measurement Models
   - Set Models

   a) Explain the difference between Region Models, Length Models and Set Models. Give examples of each model.

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   b) Why does the CAPS document say that the learners should not only work with one kind of fraction model?

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   c) What does the CAPS document suggest when using fraction models in the Intermediate Phase?

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2) Use the Clarification Notes for Grades 7, 8 and 9 CAPS in Appendix 2 to answer the following question.

   The Senior Phase CAPS does not specify that these three models should be used to develop the concept of fractions.

   a) Why do you think the three models are not specified?

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   b) Does this mean that the three models should not be used in the Senior Phase?

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1.3 <u>Five meanings of fractions</u>

Work with your group
Use APPENDIX 3 (on pages 15 to 17 in the Resources Handout) to answer the following questions.

1) This article suggests that three meanings of fractions should be developed in the Foundation and Intermediate Phase.

   a) Give a brief summary of these three meanings of fractions. Give examples to explain these meanings.

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   b) How do these meanings compare to the three different models that the Intermediate Phase CAPS says should be used to develop the concept of fractions (given in Appendix 2)?

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2) It also suggests that the last two meanings of fractions should be developed in the Senior Phase.

   a) Give as brief summary of these two meanings of fractions. Give examples to explain these meanings.

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   b) Give one topic where Meaning 4 would be used in the Senior Phase?

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   c) Give one topic where Meaning 5 would be covered in the Senior Phase?

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1.4 <u>Types of Fraction Problems</u>

Divide the class into 8 groups.
Use APPENDIX 4 (on pages 18 in the Resources Handout) to answer the following questions.
Your facilitator will give each group one of the different types of problems to work with.
(Type 9 only appears in Grade 6 and deals with calculations only, so we will leave Type 9 out of this exercise)

Learners from Grade 4 to Grade 9 have to solve fraction problems in contexts. The Intermediate Phase CAPS Document lists different Types of problems that should be dealt with in Grade 4, 5 and 6.

1) For your allocated Type, write a short description of the problems given for Grade 4, Grade 5 and Grade 6.

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2) For your allocated Type, make up your own Grade 7 example similar to the three given. Use Appendix 1 and Appendix 2 to make sure that the number range and operations asked for in your example are suitable for Grade 7.

   Write your examples on flip-chart paper and stick the paper up.

   Be prepared to explain to your colleagues how your example meets the requirements for Grade 7. This means that you need to be able to explain how it fits into the Type allocated to you and also how where it fits into the Grade 7 fraction curriculum.

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SESSION 2: USING CONCRETE APPARATUS TO TEACH FRACTIONS

2.1 <u>Concrete, Semi-Concrete and Abstract Methods of Teaching Fractions</u>

Answer question 1) with your group.
Then listen to your facilitator who will read the information given in APPENDIX 5 (on page 19 in the Resources Handout) to the class.
Then answer question 2) with your group.

1) Before listening to the reading of Appendix 5, write down what do you understand by

   a) Concrete methods of teaching fractions

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   b) Semi-concrete methods of teaching fractions

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   c) Abstract methods of teaching fractions

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2) After listening to the reading of Appendix 5, write down whether your understanding has changed or not. Did you have any misconceptions about the terms ‘concrete’, semi-concrete’ and ‘abstract’?

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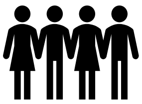
2.2 <u>Using concrete material to teach the multiplication and division of fractions</u>

NOTE:

1) Multiplication and division are dealt with in the following grades
   a) In Grade 5 the learners find fractions of whole numbers which result in whole number answers.
   b) In Grade 6 the learners find fractions of whole numbers
   c) In Grade 7 the learners
      - find fractions of whole numbers
      - multiply common fractions including mixed numbers.
   d) In Grade 8 the learners
      - find fractions of whole numbers
      - multiply common fractions including mixed numbers
      - divide whole numbers and common fractions by common fractions.
   e) In Grade 9 the learners use all four operations with common fractions and mixed numbers.

2) Because most Intermediate and Senior Phase textbooks give diagrams illustrating the addition and subtraction of fractions, but not many illustrating the multiplication and division of fractions, we will be focussing on multiplication and division here – so that you can experience what the learners are feeling when you teach them addition and subtraction of fractions.

a) MULTIPLICATION OF FRACTIONS



The Facilitator and the participants use paper folding to illustrate a concrete method of explaining the multiplication of fractions.
You then use squared paper to illustrate a semi-concrete method of explaining the multiplication of fractions.
The Participants work on Activity 1 with a partner, and this is marked.

Learners need to understand the ***conceptual ideas behind multiplication of fractions***, not simply learn the rules to follow (i.e. multiply the numerators, multiply the denominators). Using models to represent the process will help them to better understand these concepts and increase their ability to remember the rules.

- ✓ Learners know that 5 × 7 = 35 because 5 groups of seven is 35.
- ✓ An example like "$3 \times \frac{1}{2}$" is easy to understand as $3 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3\frac{1}{2}$
- ✓ When confronted with the calculation $\frac{2}{3} \times \frac{1}{4}$, most learners have difficulty understanding that the product of these numbers will be smaller than either of them. This is an expected reaction because their previous experience of multiplication (e.g. 4 × 3) makes them expect that the product will be bigger than either of the factors.

EXAMPLE 1 (Using a concrete model)

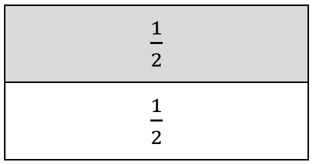
Use paper folding to determine $\frac{1}{2} \times \frac{1}{4}$, remembering that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{2}$ of $\frac{1}{4}$

Solution

STEP 1:

Use paper folding to determine $\frac{1}{2}$ of the piece of paper.

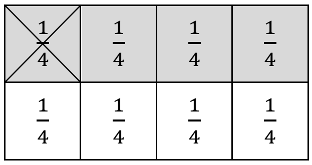
Shade in the required area



STEP 2:

Use paper folding to determine $\frac{1}{4}$ of $\frac{1}{2}$ of the sheet of paper by taking the same piece of paper and fold it in quarters (fourths) in the other direction.

Shade in $\frac{1}{4}$ of $\frac{1}{2}$ using another colour or by drawing a cross.



STEP 3:

Count up the number of equal parts (8)

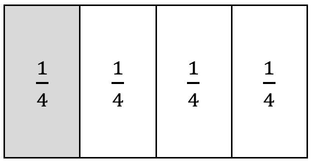
Count up the number of parts that are both shaded grey **<u>and</u>** have a cross in them (1)

$\therefore \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

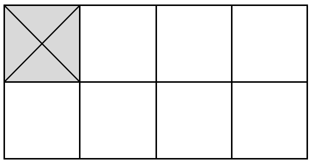
NOTE:

Because we know that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2}$ (multiplication is commutative)

We could first find $\frac{1}{4}$ of the piece of paper

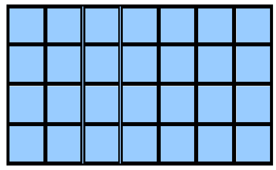


And then find $\frac{1}{2}$ of $\frac{1}{4}$ by using another colour or by drawing a cross.



NOTE:
You know that the area of a rectangle is found by multiplying the length of the one side by the length of the other side.

So, the area of this rectangle is 28 squares because the one side is 4 units and the other side is 7 units.

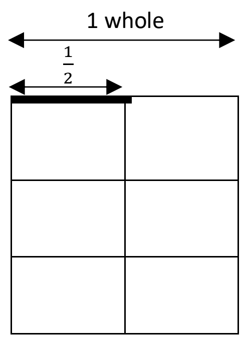


EXAMPLE 2 (Drawing a Model)

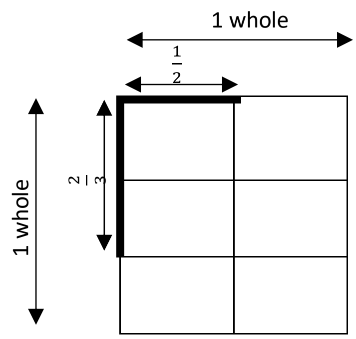
Determine $\frac{1}{2} \times \frac{2}{3}$ using the fact that ***area of a rectangle = length of the rectangle × breadth of a rectangle*** and using squared paper

Solution

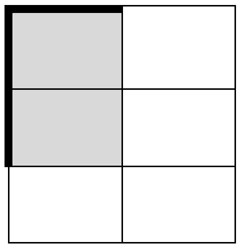
- ✓ To work out $\frac{1}{2} \times \frac{2}{3}$ start off by drawing a rectangle that is 2 units by 3 units on squared paper. (These measurements come from the denominators of the two fractions). The fractions being multiplied represent length of the sides, whereas the answer represents area.
- ✓ Follow the following procedure:
  - Divide the side that is 2 units long into halves



  - Divide the side that is 3 units long into thirds



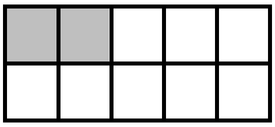
  - Shade in the area that is $\frac{1}{2}$ by $\frac{2}{3}$

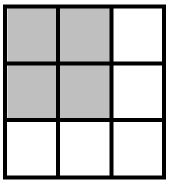


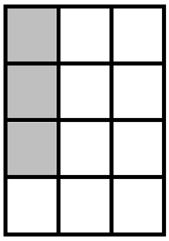
From the diagram we can see that $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

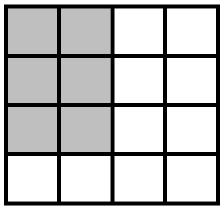
ACTIVITY 1

1) The pictures show fractional areas. Write down the multiplication sentence that goes with each one. Give the answer in simplest form.

a) 

b) 

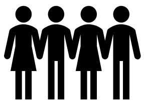
c) 

d) 

2) Draw diagrams as in the previous question to determine the answers to the following:

a) $\frac{1}{4}\times\frac{1}{2}$ b) $\frac{1}{5}\times\frac{1}{3}$ c) $\frac{5}{8}\times\frac{1}{6}$ d) $\frac{3}{4}\times\frac{5}{6}$

b) DIVISION OF FRACTIONS

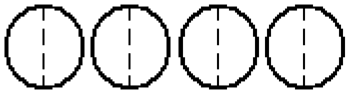


The Facilitator discusses Example 3 which illustrates semi-concrete methods of explaining the division of fraction with the Participants.
The Participants work on Activity 2 with a partner and this is marked.

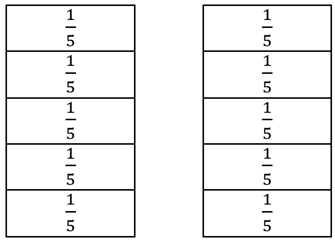
- ✓ To divide means to split into **equal** parts or groups.
- ✓ Before learners learn to *divide fractions*, they should first explore the division of whole number by a fraction and only then go onto the division of a fraction by a fraction. But, teaching learners how to divide with fractions by simply showing them how to multiply the dividend (the first number) and the reciprocal of the divisor (the second number) offers practically no insight to their mathematical understanding because no connections have been made to what they already know about division.

EXAMPLE 3 (drawing models)

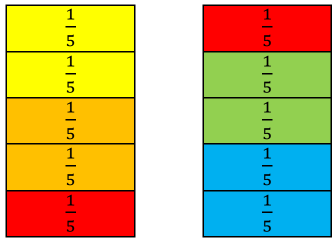
a) How many halves are there in 4?
From the diagram it can be seen that there are **eight** halves in 4.
We write this $4 \div \frac{1}{2} = 8$



b) $2 \div \frac{1}{5}$ means 'how many fifths are there in 2'
We take 2 wholes and count up the number of fifths
There are 10 fifths in 2 wholes, so $2 \div \frac{1}{5} = 10$.
We write $2 \div \frac{1}{5} = 2 \times 5 = 10$



c) $2 \div \frac{2}{5}$ means 'how many two-fifths are there in 2'
We take 2 wholes and count up the number of two-fifths
There are 5 two-fifths in 2 wholes, so $2 \div \frac{2}{5} = 5$
We write $2 \div \frac{2}{5} = \frac{2}{1} \times \frac{5}{2} = \frac{10}{2} = 5$

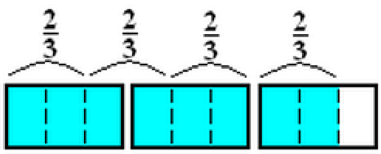


d) We can use a diagram to find the answer to $\frac{1}{2} \div \frac{1}{6}$
$\frac{1}{2} \div \frac{1}{6}$ is really asking 'how many sixths are there in $\frac{1}{2}$'?
There are 3 sixths in one-half, so $\frac{1}{2} \div \frac{1}{6} = 3$
We write $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1}$
$= \frac{6}{2}$
$= 3$

Look at the pizzas below:
How many sixths of a pizza fit into $\frac{1}{2}$ a pizza?

There are 3 sixths in $\frac{1}{2}$ a pizza

e) How many $\frac{2}{3}$ are there in $2\frac{2}{3}$?
From the diagram it can be seen that there are **four** two-thirds in $2\frac{2}{3}$.
We write $2\frac{2}{3} \div \frac{2}{3} = \frac{8}{3} \times \frac{3}{2} = \frac{24}{6} = 4$



We write this $2\frac{2}{3} \div \frac{2}{3} = 4$

ACTIVITY 2

Draw diagrams to work out

1) How many thirds there are in 3?
2) $2 \div \frac{1}{6}$
3) $5 \div \frac{5}{6}$

SOLUTION

c) USING ABSTRACT REASONING TO EXPLAIN WHY WE CHANGE DIVISION TO MULTIPLICATION AND INVERT THE SECOND NUMBER

The Facilitator illustrates abstract methods of explaining the division of fractions to the Participants.
The Participants work on Activity 3 with a partner and this is marked

✓ The learners need to understand reciprocals before they start to use them to divide fractions.

Reciprocals

Two numbers are reciprocals of each other if their product is 1. In other words, when you multiply them, you get 1.

- $\frac{4}{3}$ is a reciprocal of $\frac{3}{4}$ because $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$
- $\frac{7}{1}$ is a reciprocal of $\frac{1}{7}$ because $\frac{1}{7} \times \frac{7}{1} = \frac{7}{7} = 1$
- $\frac{1}{7}$ is a reciprocal of $\frac{7}{1}$ because $\frac{7}{1} \times \frac{1}{7} = \frac{7}{7} = 1$

✓ Dividing by a whole number can be thought of as dividing or sharing something between a certain number of people.

☞ If you divide by two, you cut something in half.
Then (for example) half of a half can be found by cutting the half into halves. The size of half of a half can be given in relation to the original whole.

☞ The following generalisation can be made:

- Dividing a number by 2 is the same as finding $\frac{1}{2}$ of the number.
- Dividing a number by 3 is the same as finding $\frac{1}{3}$ of the number.

☞ The algorithm the learners will have to learn ultimately is, instead of dividing by a fraction, multiply by its reciprocal number.

- Instead of dividing by $\frac{1}{2}$, multiply by its reciprocal 2.
- Instead of dividing by $\frac{3}{4}$, multiply by its reciprocal $\frac{4}{3}$.
- Instead of dividing by $\frac{5}{8}$, multiply by its reciprocal $\frac{8}{5}$.

EXAMPLE 4 (Why the division algorithm works)

Determine a) $\frac{1}{2} \div 2$ b) $\frac{3}{8} \div \frac{2}{5}$

Solution

a) $\frac{1}{2} \div 2 = \frac{\frac{1}{2}}{2}$ … In the first step write the division as a fraction

$= \frac{\frac{1}{2}}{2} \times \frac{\frac{1}{2}}{\frac{1}{2}}$ … We want to change the denominator into 1. We do this by multiplying by 1 or by $\frac{\frac{1}{2}}{\frac{1}{2}}$

$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{2}{1} \times \frac{1}{2}}$ … Multiply both the numerator and denominator by $\frac{1}{2}$

$= \frac{\frac{1}{4}}{1}$ … Simplify

$= \frac{1}{4}$

This is the same as multiplying by the inverse of the denominator: $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

b) $\frac{3}{8} \div \frac{2}{5} = \frac{\frac{3}{8}}{\frac{2}{5}}$ … Write the division as a fraction

$= \frac{\frac{3}{8}}{\frac{2}{5}} \times \frac{\frac{5}{2}}{\frac{5}{2}}$ … Multiply by 1. This time $1 = \frac{\frac{5}{2}}{\frac{5}{2}}$ because $\frac{5}{2}$ is the reciprocal of the $\frac{2}{5}$ in the denominator

$= \frac{\frac{3}{8} \times \frac{5}{2}}{\frac{2}{5} \times \frac{5}{2}}$

$= \frac{\frac{15}{16}}{1}$

$= \frac{15}{16}$

This is the same as multiplying by the inverse of the denominator: $\frac{3}{8} \div \frac{2}{5} = \frac{3}{8} \times \frac{5}{2} = \frac{15}{16}$

✓ Learners who use the detailed method recognise that the top fraction is being multiplied by the reciprocal of the bottom fraction. The process of discovering this ‘trick’ for themselves gives them ownership of the larger conceptual understanding. In other words, learners who learn to divide fractions in this way understand at a fundamental level ***why*** the trick works.

ACTIVITY 3

1) Do the following calculations like Example 3 *showing your solutions in full*:

   a) $\frac{12}{25} \div \frac{21}{50}$

   b) $3\frac{2}{5} \div \frac{1}{3}$

   c) $4\frac{1}{2} \div 3\frac{1}{3}$

ACTIVITY 3 (continued)

2) Look at the working presented by the following learners. In each case:
   a) Explain the mistake made by the learner
   b) Give the correct solution to each calculation.

Peter: I worked out that $\frac{1}{8} \div \frac{2}{5} = \frac{2}{40}$ $= \frac{1}{20}$	*Jabu: My calculation gives* $\frac{3}{4} \div 1\frac{1}{4} = \frac{3}{5} \times \frac{4}{1}$ $= \frac{12}{5}$ $= 2\frac{2}{5}$
Thabo: How did I go wrong? $1\frac{1}{10} \div 2\frac{1}{4} = \frac{11}{10} \times \frac{9}{4}$ $= \frac{99}{40}$	*Romy: Why doesn't this work?* $10\frac{1}{8} \div 2\frac{1}{4} = 10 \div 2 + \frac{1}{8} \div \frac{1}{4}$ $= 5\frac{1}{2}$

SESSION 3: USING MAGIC SQUARES TO PRACTISE FRACTION CALCULATIONS

3.1. What is a magic square?

The Class discusses what a Magic Square is.
The Participants work on Activity 1 with a partner, and this is marked.

A **3 by 3 Magic Square** is a square grid filled with nine distinct numbers. The sum of the numbers in ***each row*** is equal to the sum of the numbers in ***each column*** and is also equal to the sum of the numbers in the ***two main diagonals***

The following example is a magic square using fractions with identical denominators

1	$3\frac{1}{2}$	3
$4\frac{1}{2}$	$2\frac{1}{2}$	$\frac{1}{2}$
2	$1\frac{1}{2}$	4

ROWS	COLUMNS	MAIN DIAGONALS
$1 + 3\frac{1}{2} + 3 = 7\frac{1}{2}$	$1 + 4\frac{1}{2} + 2 = 7\frac{1}{2}$	$1 + 2\frac{1}{2} + 4 = 7\frac{1}{2}$
$4\frac{1}{2} + 2\frac{1}{2} + \frac{1}{2} = 7\frac{1}{2}$	$3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} = 7\frac{1}{2}$	$2 + 2\frac{1}{2} + 3 = 7\frac{1}{2}$
$2 + 1\frac{1}{2} + 4 = 7\frac{1}{2}$	$2 + 1\frac{1}{2} + 4 = 7\frac{1}{2}$	

$7\frac{1}{2}$ *is called the* ***constant*** or ***the magic total*** of *this magic square.*

Once you know what the constant or magic total is of a magic square, then it is possible to find the missing numbers.

ACTIVITY 1

- Find the Magic total for each of these Magic Squares.
- Then use addition and subtraction to find the missing numbers.

a) Identical denominators

$\frac{2}{5}$		
$1\frac{4}{5}$		
$\frac{4}{5}$		$1\frac{3}{5}$

Magic total = ……….

b) One denominator is a multiple of the other

	$1\frac{3}{4}$	$1\frac{1}{2}$
	$1\frac{1}{4}$	
1		

Magic total = ………..

3.2 How to design a 3 by 3 Magic Square

The Facilitator describes the steps to go through by writing on the board.
The Participants work on Activity 2 with their partner, and the work is marked.

STEP 1:
Decide on a sequence of ***nine*** numbers, each of which must differ from its neighbouring number by the same number. For example, use the sequence $\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$, $2\frac{2}{3}$, 3

STEP 2
Draw two grids like the following on squared paper

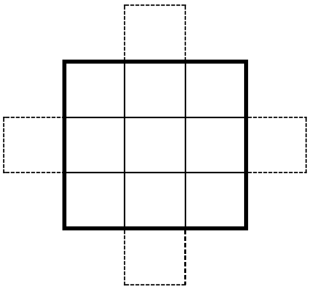


Table A

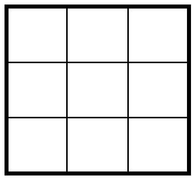


Table B

STEP 3
Write the numbers in your sequence diagonally in Table A

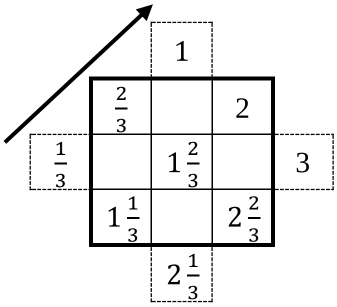


Table A

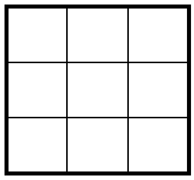


Table B

STEP 4
- Fill in all the numbers that are *inside* of the square in Table A into the same position in Table B
- Move each of the numbers that lie *outside* of the square in Table A to the *furthest* open block in its row or column in Table B.

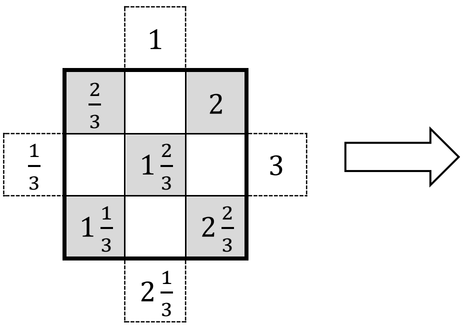


Table A

$\frac{2}{3}$	$2\frac{1}{3}$	2
3	$1\frac{2}{3}$	$\frac{1}{3}$
$1\frac{1}{3}$	1	$2\frac{2}{3}$

Table B

In other words,
- move the 1 on top of the table to below the $1\frac{2}{3}$
- move the $2\frac{1}{3}$ from the bottom of the table to above the $1\frac{2}{3}$
- move the $\frac{1}{3}$ on the left of the table to the right of $1\frac{2}{3}$
- move the 3 on the right of the table to the left of the $1\frac{2}{3}$

STEP 5
Check that the sum of the numbers in every row, every column and each of the two main diagonals is 5.

ACTIVITY 2

1) For each of the following two sequences, determine what you add on to the first term to get the second term.

   a) $\frac{1}{2}$; $\frac{2}{2}$; $\frac{3}{2}$; $\frac{4}{2}$; $\frac{5}{2}$; $\frac{6}{2}$; $\frac{7}{2}$; $\frac{8}{2}$; $\frac{9}{2}$ ………………………………………………

   b) $2\frac{1}{8}$; $2\frac{1}{4}$; $2\frac{3}{8}$; $2\frac{1}{2}$; $2\frac{5}{8}$; $2\frac{3}{4}$; $2\frac{7}{8}$; 3; $3\frac{1}{8}$ ……………………………………………….

2) Construct two 3 by 3 Magic Squares using the given sequences of numbers.
   Once you have completed the Magic Squares, check them by adding the numbers in the columns, rows and the two main diagonals.

a)

		$\frac{3}{2}$		
	$\frac{2}{2}$		$\frac{6}{2}$	
$\frac{1}{2}$		$\frac{5}{2}$		$\frac{9}{2}$
	$\frac{4}{2}$		$\frac{8}{2}$	
		$\frac{7}{2}$		

Table A

Magic Number

= …………..

b)

Table A

Magic Number

= …………….

Table B

ACTIVITY 2 (continued)

3) Make up your own sequence of fractions and mixed numbers.

……

Write the sequence of numbers in Table A. Use Table A to turn Table B into a Magic Square.

Table A

Table B

Once you have completed the magic square, check it by adding the numbers in the columns, rows and the two main diagonals.

Write the magic number here: ………………………………………………………….

3.3 <u>How to design a 4 by 4 Magic Square</u>

The Facilitator describes the steps to go through by writing on the board.
The Participants work on Activity 3 with their partner, and the work is marked.

The method used for designing a **<u>4 by 4 Magic Square</u>** is completely different to the method used for designing a 3 by 3 Magic Square.

STEP 1: Decide on a sequence of ***sixteen*** numbers, each of which differs from its neighbouring number by the same number.

e.g. Start at 0 and add $\frac{1}{6}$th to each term to get

$0;\ \frac{1}{6};\ \frac{1}{3};\ \frac{1}{2};\ \frac{2}{3};\ \frac{5}{6};\ 1;\ 1\frac{1}{6};\ 1\frac{1}{3};\ 1\frac{1}{2};1\frac{2}{3};1\frac{5}{6};2;\ 2\frac{1}{6};2\frac{1}{3};\ 2\frac{1}{2}$.

STEP 2: Write the terms of the sequence in order in the table.

0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
$\frac{2}{3}$	$\frac{5}{6}$	1	$1\frac{1}{6}$
$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{2}{3}$	$1\frac{5}{6}$
2	$2\frac{1}{6}$	$2\frac{1}{3}$	$2\frac{1}{2}$

Is this a magic square?

STEP 3: Reverse the order of the numbers in each *main diagonal*

$2\frac{1}{2}$			2
	$1\frac{2}{3}$	$1\frac{1}{2}$	
	1	$\frac{5}{6}$	
$\frac{1}{2}$			0

STEP 4: Add the missing numbers back into their original positions

$2\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	2
$\frac{2}{3}$	$1\frac{2}{3}$	$1\frac{1}{2}$	$1\frac{1}{6}$
$1\frac{1}{3}$	1	$\frac{5}{6}$	$1\frac{5}{6}$
$\frac{1}{2}$	$2\frac{1}{6}$	$2\frac{1}{3}$	0

Is this a magic square?

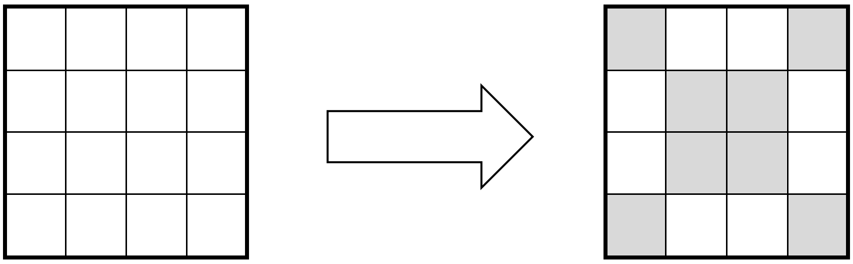
If so, what is its magic number? ………………………….

ACTIVITY 3

Use the following two sequences of numbers to construct two 4 by 4 Magic Squares. Once you have completed each Magic Square, check it by adding the numbers in the columns, rows and the two main diagonals

1) $\frac{1}{6}$; $\frac{2}{6}$; $\frac{3}{6}$; $\frac{4}{6}$; $\frac{5}{6}$; 1; $1\frac{1}{6}$; $1\frac{2}{6}$; $1\frac{3}{6}$; $1\frac{4}{6}$; $1\frac{5}{6}$; 2; $2\frac{1}{6}$; $2\frac{2}{6}$; $2\frac{3}{6}$; $2\frac{4}{6}$

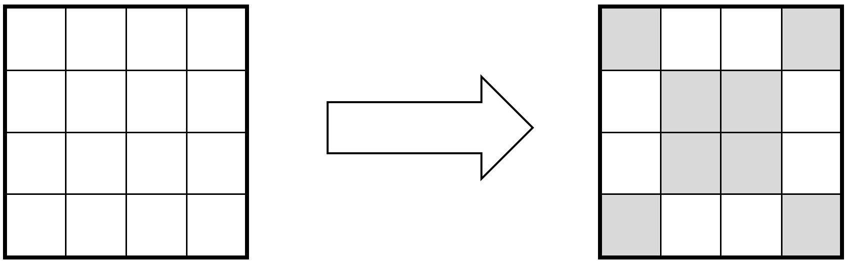
a)



Magic Number

= ………..

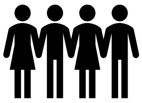
2) $\frac{1}{12}$; $\frac{2}{12}$; $\frac{3}{12}$; $\frac{4}{12}$; $\frac{5}{12}$; $\frac{6}{12}$; $\frac{7}{12}$; $\frac{8}{12}$; $\frac{9}{12}$; $\frac{10}{12}$; $\frac{11}{12}$; 1; $1\frac{1}{12}$; $1\frac{2}{12}$; $1\frac{3}{12}$; $1\frac{4}{12}$



Magic Number

= ………….

3.4 Converting your Magic Square into an Activity



The Facilitator describes the steps to go through by writing on the board.
The Participants work on Activity 4 question 1 on their own and the swap Magic Squares with a partner.
They then discuss with their partner the usefulness of Magic Squares when teaching fractions.

Magic Squares give the learners practice adding and subtracting whole numbers, fractions, decimals, and even negative numbers (integers).

In order to create a Magic Square Activity (like those in Activity 1) that you could give to the learners to solve, follow the following procedure:

STEP 1 – Create a Magic Square
Suppose you take the sequence 1; 3; 5; 7; 9; 11; 13; 15; 17

		5		
	3		11	
1		9		17
	7		15	
		13		

Table A

Becomes

3	13	11
17	9	1
7	5	15

Table B

STEP 2 – Copy either one complete row, one complete column, or one main diagonal into a blank 4 by 4 square

Example 1

3	13	11

Example 2

		11
	9	
7		

STEP 3 – Add in as few of the other numbers as possible

Example 1

3	13	11
	9	

Example 2

		11
	9	1
7		

Is it necessary to add in any other numbers or have you been given enough information to find the missing numbers?

ACTIVITY 4: Create a 3 by 3 Magic Square and turn it into an Activity

1) Create a 3 by 3 Magic Square by following the following steps:
   a) Create a 9-term sequence of fractions. (Don't use one we have used already.)

      ……………………………………………………………………………..

   b) Use the sequence to create a Magic Square on the squared paper below.
   c) Create your own Magic Square Activity by rewriting the Magic Square, removing it as many numbers as possible. Remember to leave in one column or one row or one main diagonal so that the person solving the Magic Square can work out the magic number before they find the missing terms.
   d) Give your Magic Square to one of your colleagues to complete. Make sure your cover your working out so they work out the Magic Square without clues.

2) Once you have finished this activity, discuss with your partner whether you think that you would use Magic Squares in your classroom. Do you think they are useful or not? Explain why.

   ………………………………………………………………………………………………
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