education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Grades 4 to 7
Just-in-Time Training Workshop
2019: No.1

Facilitator's Guide

Mathematics

Endorsed by:

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what I do matters

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WORKSHOP PROGRAM – GRADES 4 to 7 *(on page 1 of the Participant's Guide)*	
Session 1: 2 hours	**DEVELOPING FRACTION CONCEPTS** **1.1 Engaging with the Intermediate and Senior Phase CAPS documents** Use Appendix 1 to list the Grades where each fraction concept is introduced and taught. **1.2 Three different models used to develop the concept of fractions** Use Appendix 2 to describe the three models and to give examples of these models. **1.3 Five meanings of fractions** Use Appendix 3 to investigate the five meanings of fractions. **1.4 Types of fraction problems** Use Appendix 4 to write a short description of the Grade 4, Grade 5 and Grade 6 problems given for the Type of fraction allocated to you. Make up your own Grade 7 problem, similar to the Grade 4, 5 and 6 problems you have been allocated. Share them with the class.
Session 2: 1 hour 30 minutes	**USING CONCRETE APPARATUS TO TEACH FRACTIONS** **2.1 What is meant by concrete, semi-concrete and abstract methods of teaching?** Write down what you understand by these terms. These terms are then explained using Appendix 5 **2.2 Using Concrete and Semi-Concrete Material to teach the multiplication and division of fractions** The following topics are covered a) Multiplication of fractions b) Division of fractions c) Using abstract reasoning to explain why we change division to multiplication and invert the second number
Session 3: 1 hour 30 minutes	**USING MAGIC SQUARES TO PRACTICE FRACTION CALCULATIONS** **3.1 What is a Magic Square?** What is a Magic Square and how do we use it? **3.2 How to design a 3 × 3 Magic Square** Learn to design your own 3 × 3 Magic Square **3.3 How to design a 4 × 4 Magic Square.** Learn how to design your own 4 × 4 Magic Square **3.4 Converting your Magic Square into an Activity** Select a sequence of 9 fractions, create your own Magic Square and then turn it into an addition and subtraction activity. Discuss the usefulness of Magic Squares.

NOTE TO THE FACILITATOR FOR SESSION 1

(allow 2 hours for this session)

RESOURCES:

- Each educator will need a Pre-Test, a Participant's Handout and the Resources Material
- Each pair will need one sheet of flipchart paper and a marker pen for *1.4 TYPES OF FRACTION PROBLEMS*

PRE-TEST

25 minutes	*Each educator writes the test on the question paper. Take the papers in.*

ORGANISE THE EDUCATORS INTO GROUPS

Before you start, make sure the Participants are sitting in groups.

To form groups:

STEP 1:	*Decide on the size of your group (5 or 6 in a group is a good size)*
STEP 2:	*Mentally divide the number of Participants in your group by 5 (or 6).* *Suppose you have 30 Participants. $30 \div 5 = 6$. So, you will have 6 groups of 5.*
STEP 3:	*Because there are going to be 6 groups, go around the class, numbering the Participants from 1 to 6.* *Tell the Participants to remember their numbers (NOTE: they are bad at this!)*
STEP 4:	*Tell the 1s to form a group, the 2s to form a group, and so on.* *If they have remembered their numbers, you should have 6 groups of equal size.* *If you don't have groups of equal size, move Participants around to get equal-sized groups.*

1.1: ENGAGING WITH THE INTERMEDIATE AND SENIOR PHASE CAPS

25 minutes	*Working with their group, the Participants use Appendix 1 given in the Resource Handout to list the Grades where each fraction concept is introduced and taught.* • *Discuss the answers with the Participants.*

1.2: THREE DIFFERENT MODELS USED TO DEVELOP THE CONCEPT OF FRACTIONS

25 minutes	*Working with their group, the Participants use Appendix 2 given in the Resource Handout to investigate the 3 models listed in the Intermediate Phase CAPS and discuss why these 3 models are not given in the Senior Phase CAPS.* • *Discuss the answers with the Participants.*

1.3: FIVE MEANINGS OF FRACTIONS

25 minutes	*Working with their group, the Participants use Appendix 3 given in the Resource Handout to investigate five different meanings of fractions as given in an article found on the internet. Three of these meanings are relevant to the Foundation and Intermediate Phases and two are relevant to the Senior Phase.* • *Discuss the answers with the Participants.*

1.4: TYPES OF FRACTION PROBLEMS

45 minutes	*Divide the class up into 8 different groups.* *Each group has to write a short description of the types of fraction problems for Grade 4, 5 and 6 and then make up a Grade 7 problem similar to the Grade 4, 5 and 6 problems they have been allocated.* *The examples are stuck up around the classroom and the class moves around interrogating 'the person on duty'.* • *Discuss the examples with the Participants.*

SESSION 1: DEVELOPING FRACTION CONCEPTS

1.1. Engaging with the Intermediate and Senior Phase CAPS documents (p 2)

Work with your group
Use APPENDIX 1 (on pages 1 to 3 in the Resources Handout) to answer the following questions.

1) Both the Intermediate and Senior Phase CAPS documents list, for each grade, the concepts that are different to the previous grade.
   What concepts are new to the learners in

   a) Grade 4?

      Sevenths

      Addition of fractions with the same denominators

   b) Grade 5?

      Ninths, tenths, elevenths and twelfths

      Counting in fractions

      Subtraction of fractions with the same denominators

      Addition and subtraction of mixed numbers

      Fractions of whole numbers with whole number answers

   c) Grade 6?

      They name, order and compare all common fractions

      Tenths and hundredths

      Addition and subtraction of fractions where one denominator is a multiple of another

      Fractions of whole numbers with fractional answers

      Decimals

      Percentages

      Equivalence between fractions, decimals and percentages

   d) Grade 7?

      Hundredths

      Multiplication of common fractions

      Percentage of a whole

      Percentage increase and decrease

e) Grade 8?

Division of common fractions

Squares, cubes, square roots and cube roots of common fractions

f) Grade 9?

Learners consolidate number knowledge and calculation techniques for common fractions

2) In which Grades do the learners ‘Describe and Order Fractions’? ***Grades 4 to 6***

3) Study the references to Addition and Subtraction in the Phase Overview. In which Grades do the learners
   a) Start adding fractions with the same denominators? ***Grade 4***

   b) Start subtracting fractions with the same denominator? ***Grade 5***

   c) Start adding and subtracting fractions where one denominator is a multiple of another? ***Grade 6***

   d) Start adding and subtracting fractions where one denominator is NOT a multiple of the other? ***Gr 7***

   e) Start working with the addition and subtraction of mixed numbers? ***Grade 5***

4) Study the references to the Equivalence of Division and Fractions
   a) What do we mean by the equivalence of Division and Fractions? ***That*** $\mathbf{1 \div 5 = \frac{1}{5}}$
   b) In which Grades do the learners ‘recognise, describe and use the equivalence of division and fractions’? ***Grades 4 and 5***

5) Study the references to Multiplication and Division in the Phase Overview. In which Grades do the learners
   a) Find the fractions of whole numbers which result in whole number answers? ***Grades 5 to 9***

   b) Find the fractions of whole numbers which do not necessarily result in whole number answers? ***Grades 6 to 9***

   c) Multiply common fractions and mixed numbers? ***Grades 7, 8 and 9***

   d) Divide fractions? ***Grades 8 and 9***

6) In which Grades do the learners calculate the squares, cubes, square roots and cube roots of common fractions? ***Grades 8 and 9***

7) In which Grades do the learners solve problems in contexts involving fractions? ***Grades 4 to 9***

8) Study the references to Percentages. In which Grade do the learners
   a) Start working with percentages? ***Grade 6***

   b) Find the percentages of whole numbers? ***Grades 6 to 9***

   c) Calculate the percentage of parts of a whole? ***Grades 7 to 9***

   d) Calculate percentage increase or decrease? ***Grades 7 to 9***

   e) Calculate amounts if given percentage increase or decrease? ***Grades 8 and 9***

   f) Solve problems in contexts involving percentages? ***Grades 7, 8 and 9***

9) Study the references to Equivalent Forms in the Phase Overview. In which Grades do the learners
   a) Recognise and use equivalent forms of common fractions where one denominator is a multiple of another? ***Grades 4 to 9***

   b) Recognise the equivalence between common fractions, decimal fractions and percentage forms of the same number? ***Grades 6 to 9***

1.2. <u>Three different models used to develop the concept of fractions (p 5)</u>

Work with your group
Use APPENDIX 2 (on pages 4 to 14 in the Resources Handout) to answer the following questions.

1) The Clarification Notes for Grade 4, for Grade 5 and for Grade 6 CAPS state that learners should work with apparatus and diagrams to develop different ways of thinking about fractions. These different models are
   - Region or Area Models
   - Length or Measurement Models
   - Set Models

   a) Explain the difference between Region Models, Length Models and Set Models. Give examples of each model.

      <u>Region or Area Models</u> are objects that are divided into equal parts.
      They develop the concept of fractions as part of a whole and also fractions as a measure.

      <u>Examples of area models</u>: circles cut into fraction pieces or diagrams of pies, rectangles or other geometric shapes divided into fraction pieces (paper folding), fractions using square or dotty paper, geoboards

      <u>Length or Measurement Models</u> that are divided along a straight line.
      They also develop the concept of fractions as part of a whole and also fractions as a measure.

      <u>Examples of length models</u> include fraction strips, Cuisenaire rods and number lines

      <u>Set models</u> consist of a collection of objects.
      They can form the basis of thinking about a fraction of a number e.g. $\frac{1}{3}$ of 12.

      <u>Examples of set models</u> include counters of any kind in different arrangements

   b) Why does the CAPS document say that the learners should not only work with one kind of fraction model?
      Working with one kind of fraction model can limit their understanding of fractions.

   c) What does the CAPS document suggest when using fraction models in the Intermediate Phase?
      The CAPS document suggests that all three models should be used when dealing with each fractions concept in the Intermediate Phase.

2) Use the Clarification Notes for Grades 7, 8 and 9 CAPS in Appendix 2 to answer the following question.

The Senior Phase CAPS does not specify that these three models should be used to develop the concept of fractions.

a) Why do you think the three models are not specified?

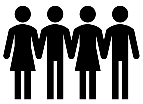
The Senior Phase states that the focus is on context free calculations which means that it is not necessary to use diagrams.

b) Does this mean that the three models should not be used in the Senior Phase?

It is assumed that the learners fraction concepts have been developed to such a stage that using diagrams to illustrate fraction calculations should not be necessary. HOWEVER, all three types of diagrams should be used where a learner is still struggling with fractions.

In the Senior Phase the learners solve problems in contexts, and one of the problem-solving techniques they could use is 'drawing a diagram'. So they could use the three fraction models to help them solve problems.

1.3 <u>Five meanings of fractions (p 7)</u>



Work with your group
Use APPENDIX 3 (on pages 15 to 17 in the Resources Handout) to answer the following questions.

1) This article suggests that three meanings of fractions should be developed in the Foundation and Intermediate Phase.

a) Give a brief summary of these three meanings of fractions. Give examples to explain these meanings.

The <u>cut or part of a whole meaning</u> involves the imagined or actual action of cutting a whole number into equal parts. This could involve cutting a quantity according to its length, area, volume or mass. For example, you could cut a piece of string into 4 equal parts; each part is $\frac{1}{4}$ of the whole; you could cut a circle into halves; you could cut a loaf of bread into 8 parts having the same volume; each part is $\frac{1}{8}$ of the volume of the loaf of bread

The <u>part of a group or set meaning</u> involves selecting objects from a group. For example, if 14 of 23 books are novels, then we know that $\frac{14}{23}$ of the books are novels.

The <u>name for a point meaning</u> involves associating marks on measuring devices with fraction names. It is also called the measure meaning of fractions. For example, a number line marked in quarters can be used as a ruler for measuring length using fractional parts of the unit of length

b) How do these meanings compare to the three different models that the Intermediate Phase CAPS says should be used to develop the concept of fractions (given in Appendix 2)?

The three models in Appendix 3 are very similar to the three models in Appendix 2. The order is just different.

2) It also suggests that the last two meanings of fractions should be developed in the Senior Phase.

a) Give as brief summary of these two meanings of fractions. Give examples to explain these meanings.

The ratio meaning** involves the comparison between two quantities.* $\frac{2}{3}$ ***implies the ratio of 2 is to 3. For example, suppose the ratio between flour and butter in a recipe is 5 cups flour to 3 tablespoons butter. The ratio is 5 : 3 or $\frac{5}{3}$.

Indicated division** meaning refers to the fact that a fraction is another way of dividing numbers. For example,* $\frac{1}{2} = 1 \div 2 = 0,5$ ***or $\frac{3}{4} = 3 \div 4 = 0,75$

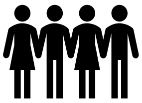
b) Give one topic where Meaning 4 would be used in the Senior Phase?

The ratio meaning would be covered when the Senior Phase learners deal with Ratio and Proportion.

c) Give one topic where Meaning 5 would be covered in the Senior Phase?

The indicated division meaning would be covered when the Senior Phase learners convert between fractions and decimals.

1.4 <u>Types of Fraction Problems (p 9)</u>



Divide the class into 8 groups.
Use APPENDIX 4 (on pages 18 in the Resources Handout) to answer the following questions.
Your facilitator will give each group one of the different types of problems to work with.
(Type 9 only appears in Grade 6 and deals with calculations only, so we will leave Type 9 out of this exercise)

Learners from Grade 4 to Grade 9 have to solve fraction problems in contexts. The Intermediate Phase CAPS Document lists different Types of problems that should be dealt with in Grade 4, 5 and 6.

1) For your allocated Type, write a short description of the problems given for Grade 4, Grade 5 and Grade 6.

TYPE 1: whole is a single object
- ***GRADE 4: one fraction of a chocolate bar; subtraction***
- ***GRADE 5: two fractions of a chocolate cake; addition and subtraction***
- ***GRADE 6: one fraction of a chocolate bar; subtraction and division***

TYPE 2: whole is a collection of objects
- ***GRADE 4: sharing of chocolates where one chocolate has to be cut up***
- ***GRADE 5: find the fraction of wall panels; subtraction***
- ***GRADE 6: fraction of the number of hours; addition and subtraction***

TYPE 3: relationship
- ***GRADE 4: find a unit fraction of an amount of money that is divisible by the denominator***
- ***GRADE 5: find a unit fraction of an amount of money that is not divisible by the denominator***
- ***GRADE 6: find a non-unit fraction of an amount of money***

TYPE 4: ratio
- ***GRADE 4: given the amount of milk needed for one batch of biscuits, find the amount needed for 5 batches***
- ***GRADE 5: given the ratio of sugar to butter for one batch, use the ratio to find the amount of butter needed***
- ***GRADE 6: given the amount of milk needed for 40 biscuits, determine the amount of milk needed for 1 000 biscuits***

TYPE 5: comparator
- ***GRADE 4: comparing two fractions of a quantity***
- ***GRADE 5: comparing two equivalent fractions of a quantity***
- ***GRADE 6: comparing fractions of a metre using division by 100 and***

TYPE 6: unit of measurement
- ***GRADE 4: subtracting fractions of a metre where both fractions have the same denominator***
- ***GRADE 5: repeated subtraction of fractions of a metre where both fractions have the same denominator***
- ***GRADE 6: repeated subtraction of fractions of a metre where the fractions have different denominators***

TYPE 7: number

- ***GRADE 4: find a fraction between a mixed number and a whole number***
- ***GRADE 5: find a fraction between two mixed numbers where the fractions have different denominators***
- ***GRADE 6: placing fractions and mixed numbers with different denominators on a number line***

TYPE 8: fractional parts put together to make a whole

- ***GRADE 4: given a simple fraction of an orange given to one athlete*** $\left(\text{here } \frac{1}{2}\right)$***, find out how many is needed for 55 athletes***
- ***GRADE 5: given the fraction of a bottle of cool drink given to one person, find out how much is needed for 35 children***
- ***GRADE 6: given the fraction of a bottle of cool drink and fraction of a bar of chocolate given to one person, find out how much is needed for 500 children***

2) For your allocated Type, make up your own Grade 7 example similar to the three given. Use Appendix 1 and Appendix 2 to make sure that the number range and operations asked for in your example are suitable for Grade 7.

   Write your examples on flip-chart paper and stick the paper up.

   Be prepared to explain to your colleagues how your example meets the requirements for Grade 7. This means that you need to be able to explain how it fits into the Type allocated to you and also how where it fits into the Grade 7 fraction curriculum.

Answers will differ.

One person from the pair remains at their example and is the person 'on duty'.
The rest of the class walks around looking at the answers and asking questions of the person 'on duty'.
The person on duty must be prepared to explain their examples to whoever asks and must also be able to explain where the example it fits in the Gr 7 fraction curriculum.

NOTE TO THE FACILITATOR FOR SESSION 2

(allow 90 minutes ($1\frac{1}{2}$ hours) for this session)

RESOURCES

- Each participant needs 2 sheets of scrap paper

2.1: WHAT IS MEANT BY CONCRETE, SEMI-CONCRETE AND ABSTRACT METHODS OF TEACHING?

30 minutes	*The participants write down what they understand by concrete, semi-concrete and abstract methods of teaching fractions.* *The facilitator uses Appendix 5 (on page 20 in the Resources Handout) to explain to the Participants the difference between concrete, semi-concrete and abstract phases of teaching.* *The participants write down whether their understanding of the terms has changed or not.*

2.2: USING CONRETE AND SEMI-CONCRETE MATERIAL TO TEACH THE MULTIPLICATION AND DIVISION OF FRACTIONS

20 minutes	***a) Multiplication of Fractions*** *Example 1: Use paper folding to determine* $\frac{1}{2} \times \frac{1}{4}$ *Example 2: Draw a model to determine* $\frac{1}{2} \times \frac{2}{3}$ *Complete Activity 1 where the Participants have to interpret multiplication diagrams and draw their own multiplication diagrams*
20 minutes	***b) Division of Fractions*** *Discuss the 5 examples which use diagrams to explain division* *Complete Activity 2 where the Participants draw diagrams to work out the answer to division examples*
20 minutes	***c) Using Abstract Reasoning to explain why we change division to multiplication and invert the second number*** *Discuss reciprocals and the meaning of division* *Example 3: Use mathematical methods to divide by a fraction* *Complete Activity 3 where the Participants find the answers to the division of fractions by writing down every step and then analyse where 4 learners went wrong and then give the correct answer.*

SESSION 2: USING CONCRETE APPARATUS TO TEACH FRACTIONS

2.1 Concrete, Semi-Concrete and Abstract Methods of Teaching Fractions (p 11)

Answer question 1) with your group.
Then listen to your facilitator who will read the information given in APPENDIX 5 (on page 20 in the Resources Handout) to the class.
Then answer question 2) with your group.

1) Before listening to the reading of Appendix 5, write down what do you understand by

   a) Concrete methods of teaching fractions

      Answers will differ

   b) Semi-concrete methods of teaching fractions

      Answers will differ

   c) Abstract methods of teaching fractions

      Answers will differ

2) After listening to the reading of Appendix 5, write down whether your understanding has changed or not. Did you have any misconceptions about the terms ‘concrete’, semi-concrete’ and ‘abstract’?

2.2 Using concrete material to teach the multiplication and division of fractions (p 12)

NOTE:

1) Multiplication and division are dealt with in the following grades
   a) In Grade 5 the learners find fractions of whole numbers which result in whole number answers.
   b) In Grade 6 the learners find fractions of whole numbers
   c) In Grade 7 the learners
      - find fractions of whole numbers
      - multiply common fractions including mixed numbers.
   d) In Grade 8 the learners
      - find fractions of whole numbers
      - multiply common fractions including mixed numbers
      - divide whole numbers and common fractions by common fractions.
   e) In Grade 9 the learners use all four operations with common fractions and mixed numbers.

2) Because most Intermediate and Senior Phase textbooks give diagrams illustrating the addition and subtraction of fractions, but not many illustrating the multiplication and division of fractions, we will be focussing on multiplication and division here – so that you can experience what the learners are feeling when you teach them addition and subtraction of fractions.

a) MULTIPLICATION OF FRACTIONS

The Facilitator and the participants use paper folding to illustrate a concrete method of explaining the multiplication of fractions.
You then use squared paper to illustrate a semi-concrete method of explaining the multiplication of fractions.
The Participants work on Activity 1 with a partner, and this is marked.

Learners need to understand the ***conceptual ideas behind multiplication of fractions***, not simply learn the rules to follow (i.e. multiply the numerators, multiply the denominators). Using models to represent the process will help them to better understand these concepts and increase their ability to remember the rules.

- ✓ Learners know that 5 × 7 = 35 because 5 groups of seven is 35.
- ✓ An example like "$3 \times \frac{1}{2}$" is easy to understand as $3 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3\frac{1}{2}$
- ✓ When confronted with the calculation $\frac{2}{3} \times \frac{1}{4}$, most learners have difficulty understanding that the product of these numbers will be smaller than either of them. This is an expected reaction because their previous experience of multiplication (e.g. 4 × 3) makes them expect that the product will be bigger than either of the factors.

EXAMPLE 1 (Using a concrete model)

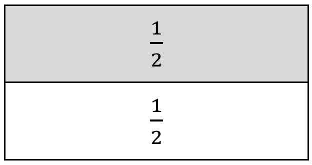
Use paper folding to determine $\frac{1}{2} \times \frac{1}{4}$, remembering that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{2}$ of $\frac{1}{4}$

Solution

STEP 1:

Use paper folding to determine $\frac{1}{2}$ of the piece of paper.

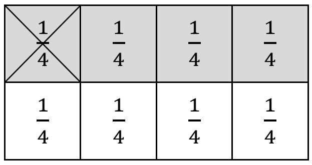
Shade in the required area



STEP 2:

Use paper folding to determine $\frac{1}{4}$ of $\frac{1}{2}$ of the sheet of paper by taking the same piece of paper and fold it in quarters (fourths) in the other direction.

Shade in $\frac{1}{4}$ of $\frac{1}{2}$ using another colour or by drawing a cross.



STEP 3:

Count up the number of equal parts (8)

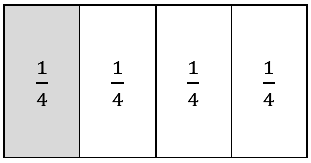
Count up the number of parts that are both shaded grey **<u>and</u>** have a cross in them (1)

$\therefore \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

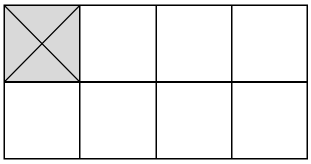
NOTE:

Because we know that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2}$ (multiplication is commutative)

We could first find $\frac{1}{4}$ of the piece of paper

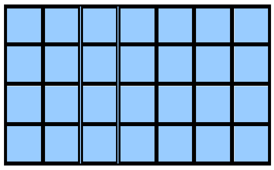


And then find $\frac{1}{2}$ of $\frac{1}{4}$ by using another colour or by drawing a cross.



NOTE:
You know that the area of a rectangle is found by multiplying the length of the one side by the length of the other side.

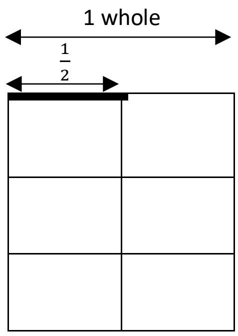
So, the area of this rectangle is 28 squares because the one side is 4 units and the other side is 7 units.



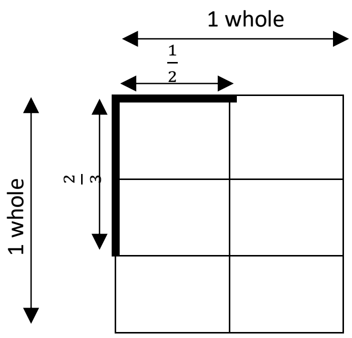
EXAMPLE 2 (Drawing a Model)
Determine $\frac{1}{2} \times \frac{2}{3}$ using the fact that ***area of a rectangle = length of the rectangle × breadth of a rectangle*** and using squared paper

Solution

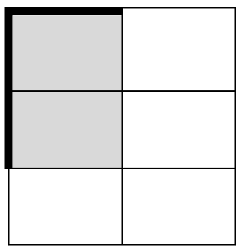
- ✓ To work out $\frac{1}{2} \times \frac{2}{3}$ start off by drawing a rectangle that is 2 units by 3 units on squared paper. (These measurements come from the denominators of the two fractions). The fractions being multiplied represent length of the sides, whereas the answer represents area.
- ✓ Follow the following procedure:
  - Divide the side that is 2 units long into halves



  - Divide the side that is 3 units long into thirds



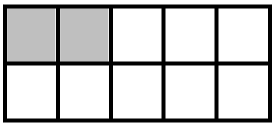
  - Shade in the area that is $\frac{1}{2}$ by $\frac{2}{3}$

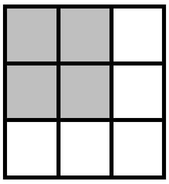


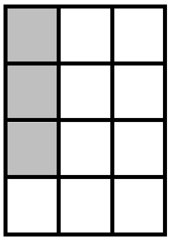
From the diagram we can see that $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

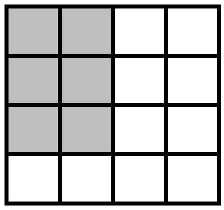
ACTIVITY 1

1) The pictures show fractional areas. Write down the multiplication sentence that goes with each one. Give the answer in simplest form.

a) 

b) 

c) 

d) 

SOLUTION

a) $\frac{2}{5} \times \frac{1}{2} = \frac{2}{10} = \frac{1}{5}$

b) $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

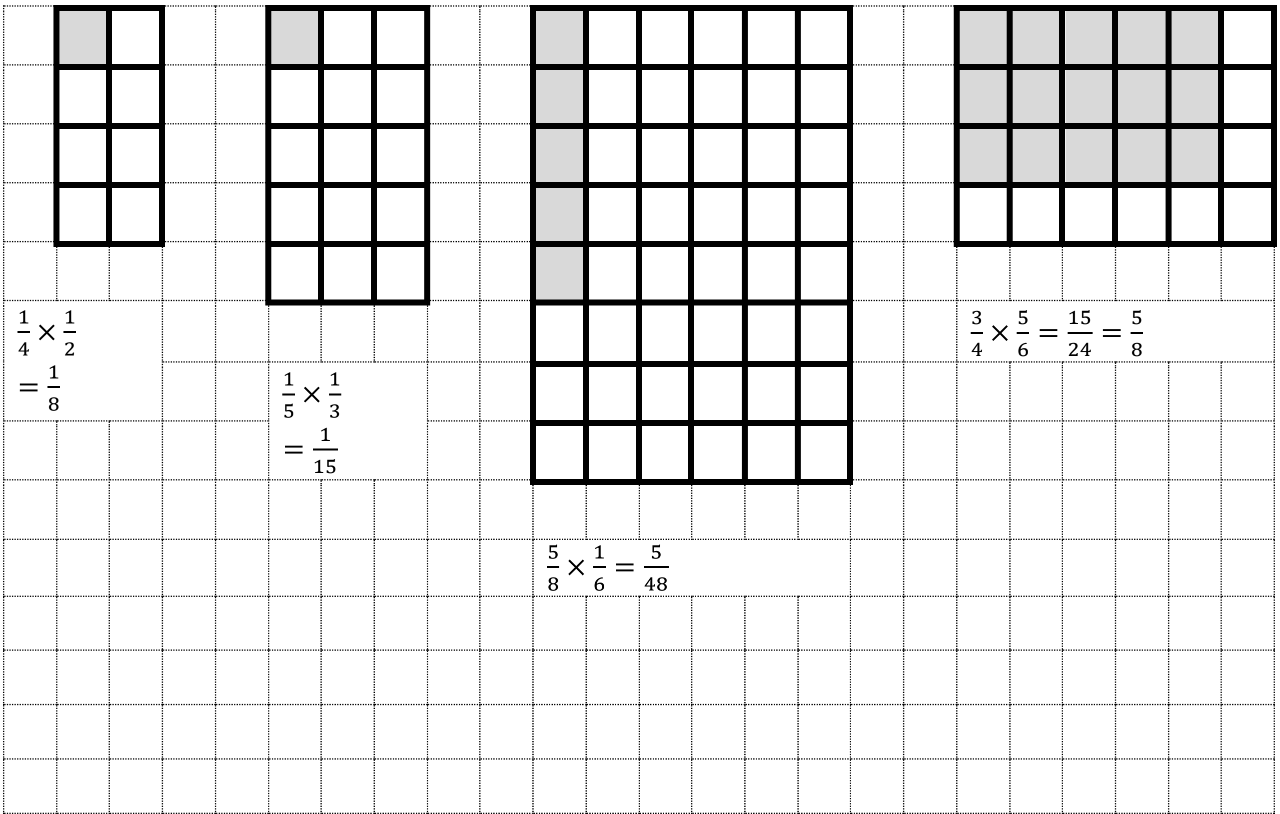
c) $\frac{1}{3} \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$

d) $\frac{3}{4} \times \frac{2}{4} = \frac{6}{16} = \frac{3}{8}$

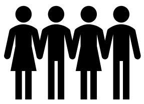
2) Draw diagrams as in the previous question to determine the answers to the following:

a) $\frac{1}{4} \times \frac{1}{2}$ b) $\frac{1}{5} \times \frac{1}{3}$ c) $\frac{5}{8} \times \frac{1}{6}$ d) $\frac{3}{4} \times \frac{5}{6}$

SOLUTION



b) DIVISION OF FRACTIONS (p 16)

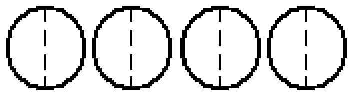


The Facilitator discusses Example 3 which illustrates semi-concrete methods of explaining the division of fraction with the Participants.
The Participants work on Activity 2 with a partner and this is marked.

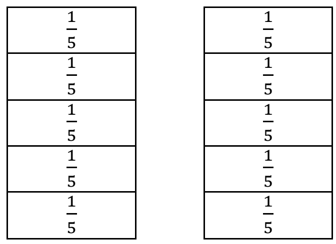
- ✓ To divide means to split into **equal** parts or groups.
- ✓ Before learners learn to *divide fractions*, they should first explore the division of whole number by a fraction and only then go onto the division of a fraction by a fraction. But, teaching learners how to divide with fractions by simply showing them how to multiply the dividend (the first number) and the reciprocal of the divisor (the second number) offers practically no insight to their mathematical understanding because no connections have been made to what they already know about division.

EXAMPLE 3 (drawing models)

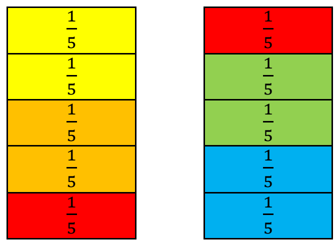
a) How many halves are there in 4?
From the diagram it can be seen that there are **eight** halves in 4.
We write this $4 \div \frac{1}{2} = 8$



b) $2 \div \frac{1}{5}$ means 'how many fifths are there in 2'
We take 2 wholes and count up the number of fifths
There are 10 fifths in 2 wholes, so $2 \div \frac{1}{5} = 10$.
We write $2 \div \frac{1}{5} = 2 \times 5 = 10$



c) $2 \div \frac{2}{5}$ means 'how many two-fifths are there in 2'
We take 2 wholes and count up the number of two-fifths
There are 5 two-fifths in 2 wholes, so $2 \div \frac{2}{5} = 5$
We write $2 \div \frac{2}{5} = \frac{2}{1} \times \frac{5}{2} = \frac{10}{2} = 5$

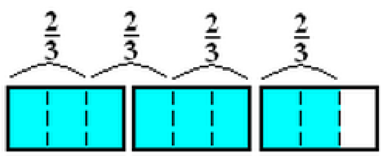


d) We can use a diagram to find the answer to $\frac{1}{2} \div \frac{1}{6}$
$\frac{1}{2} \div \frac{1}{6}$ is really asking 'how many sixths are there in $\frac{1}{2}$'?
There are 3 sixths in one-half, so $\frac{1}{2} \div \frac{1}{6} = 3$
We write $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1}$
$= \frac{6}{2}$
$= 3$

Look at the pizzas below:
How many sixths of a slice fit into $\frac{1}{2}$ a slice of pizza?

There are 3 sixths in $\frac{1}{2}$ a pizza

e) How many $\frac{2}{3}$ are there in $2\frac{2}{3}$?
From the diagram it can be seen that there are **four** two-thirds in $2\frac{2}{3}$.
We write $2\frac{2}{3} \div \frac{2}{3} = \frac{8}{3} \times \frac{3}{2} = \frac{24}{6} = 4$



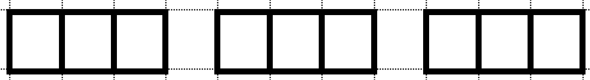
We write this $2\frac{2}{3} \div \frac{2}{3} = 4$

ACTIVITY 2

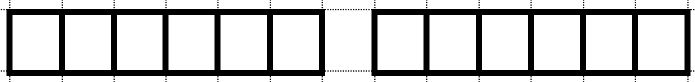
Draw diagrams to work out

1) How many thirds there are in 3?
2) $2 \div \frac{1}{6}$
3) $5 \div \frac{5}{6}$

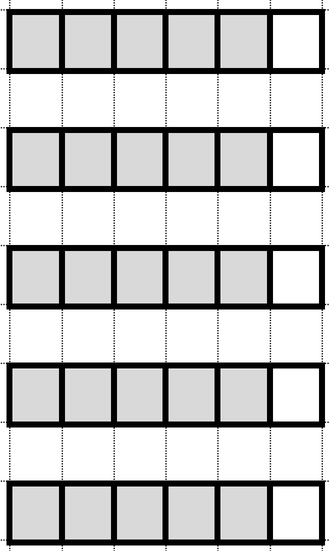
SOLUTION

1) 

There are 9 thirds in 3

2) 

$2 \div \frac{1}{6} = 12$

3) 

$5 \div \frac{5}{6} = 5$ lots of $\frac{5}{6}$ plus 5 lots of $\frac{1}{6}$ = 5 lots of $\frac{5}{6}$ + 1 lot of $\frac{5}{6}$ = 6

c) USING ABSTRACT REASONING TO EXPLAIN WHY WE CHANGE DIVISION TO MULTIPLICATION AND INVERT THE SECOND NUMBER (p 18)

The Facilitator illustrates abstract methods of explaining the division of fractions to the Participants.
The Participants work on Activity 3 with a partner and this is marked

✓ The learners need to understand reciprocals before they start to use them to divide fractions.

Reciprocals

Two numbers are reciprocals of each other if their product is 1. In other words, when you multiply them, you get 1.

- $\frac{4}{3}$ is a reciprocal of $\frac{3}{4}$ because $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$
- $\frac{7}{1}$ is a reciprocal of $\frac{1}{7}$ because $\frac{1}{7} \times \frac{7}{1} = \frac{7}{7} = 1$
- $\frac{1}{7}$ is a reciprocal of $\frac{7}{1}$ because $\frac{7}{1} \times \frac{1}{7} = \frac{7}{7} = 1$

✓ Dividing by a whole number can be thought of as dividing or sharing something between a certain number of people.

☞ If you divide by two, you cut something in half.
Then (for example) half of a half can be found by cutting the half into halves. The size of half of a half can be given in relation to the original whole.

☞ The following generalisation can be made:

- Dividing a number by 2 is the same as finding $\frac{1}{2}$ of the number.
- Dividing a number by 3 is the same as finding $\frac{1}{3}$ of the number.

☞ The algorithm the learners will have to learn ultimately is, instead of dividing by a fraction, multiply by its reciprocal number.

- Instead of dividing by $\frac{1}{2}$, multiply by its reciprocal 2.
- Instead of dividing by $\frac{3}{4}$, multiply by its reciprocal $\frac{4}{3}$.
- Instead of dividing by $\frac{5}{8}$, multiply by its reciprocal $\frac{8}{5}$.

EXAMPLE 4 (Why the division algorithm works)

Determine a) $\frac{1}{2} \div 2$ b) $\frac{3}{8} \div \frac{2}{5}$

Solution

a) $\frac{1}{2} \div 2 = \frac{\frac{1}{2}}{2}$ … In the first step write the division as a fraction

$= \frac{\frac{1}{2}}{2} \times \frac{\frac{1}{2}}{\frac{1}{2}}$ … We want to change the denominator into 1. We do this by multiplying by 1 or by $\frac{\frac{1}{2}}{\frac{1}{2}}$

$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{2}{1} \times \frac{1}{2}}$ … Multiply both the numerator and denominator by $\frac{1}{2}$

$= \frac{\frac{1}{4}}{1}$ … Simplify

$= \frac{1}{4}$

This is the same as multiplying by the inverse of the denominator: $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

b) $\frac{3}{8} \div \frac{2}{5} = \frac{\frac{3}{8}}{\frac{2}{5}}$ … Write the division as a fraction

$= \frac{\frac{3}{8}}{\frac{2}{5}} \times \frac{\frac{5}{2}}{\frac{5}{2}}$ … Multiply by 1. This time $1 = \frac{\frac{5}{2}}{\frac{5}{2}}$ because $\frac{5}{2}$ is the reciprocal of the $\frac{2}{5}$ in the denominator

$= \frac{\frac{3}{8} \times \frac{5}{2}}{\frac{2}{5} \times \frac{5}{2}}$

$= \frac{\frac{15}{16}}{1}$

$= \frac{15}{16}$

This is the same as multiplying by the inverse of the denominator: $\frac{3}{8} \div \frac{2}{5} = \frac{3}{8} \times \frac{5}{2} = \frac{15}{16}$

- ✓ Learners who use the detailed method recognise that the top fraction is being multiplied by the reciprocal of the bottom fraction. The process of discovering this ‘trick’ for themselves gives them ownership of the larger conceptual understanding. In other words, learners who learn to divide fractions in this way understand at a fundamental level ***why*** the trick works.

ACTIVITY 3

1) Do the following calculations like Example 3 *showing your solutions in full*:

a) $\frac{12}{25} \div \frac{21}{50}$

SOLUTION: $\frac{12}{25} \div \frac{21}{50} = \frac{\frac{12}{25}}{\frac{21}{50}} \times \frac{\frac{50}{21}}{\frac{50}{21}} = \frac{\frac{12}{25}\times\frac{50}{21}}{\frac{21}{50}\times\frac{50}{21}} = \frac{\frac{8}{7}}{1} = \frac{8}{7}$

b) $3\frac{2}{5} \div \frac{1}{3}$

SOLUTION: $3\frac{2}{5} \div \frac{1}{3} = \frac{\frac{17}{5}}{\frac{1}{3}} \times \frac{\frac{3}{1}}{\frac{3}{1}} = \frac{\frac{17}{5}\times\frac{3}{1}}{\frac{1}{3}\times\frac{3}{1}} = \frac{\frac{51}{5}}{1} = \frac{51}{5} = 10\frac{1}{5}$

c) $4\frac{1}{2} \div 3\frac{1}{3}$

SOLUTION: $4\frac{1}{2} \div 3\frac{1}{3} = \frac{\frac{9}{2}}{\frac{10}{3}} \times \frac{\frac{3}{10}}{\frac{3}{10}} = \frac{\frac{9}{2}\times\frac{3}{10}}{\frac{10}{3}\times\frac{3}{10}} = \frac{\frac{27}{20}}{1} = \frac{27}{20} = 1\frac{7}{20}$

ACTIVITY 3 (continued)

2) Look at the working presented by the following learners. In each case:
   a) Explain the mistake made by the learner
   b) Give the correct solution to each calculation.

Peter: I worked out that $\frac{1}{8} \div \frac{2}{5} = \frac{2}{40}$ $= \frac{1}{20}$	*Jabu: My calculation gives* $\frac{3}{4} \div 1\frac{1}{4} = \frac{3}{5} \times \frac{4}{1}$ $= \frac{12}{5}$ $= 2\frac{2}{5}$
Thabo: How did I go wrong? $1\frac{1}{10} \div 2\frac{1}{4} = \frac{11}{10} \times \frac{9}{4}$ $= \frac{99}{40}$	*Romy: Why doesn't this work?* $10\frac{1}{8} \div 2\frac{1}{4} = 10 \div 2 + \frac{1}{8} \div \frac{1}{4}$ $= 5\frac{1}{2}$

SOLUTION

<u>Peter</u>

a) Peter changed division to multiplication but didn't invert the second fraction.

b) $\frac{1}{8} \div \frac{2}{5} = \frac{1}{8} \times \frac{5}{2} = \frac{5}{16}$

<u>Jabu</u>

a) Jabu added 1 to the 4 in the denominator of $\frac{3}{4}$ to get $\frac{3}{5}$ and changed to multiplication and inverted the $\frac{1}{4}$ only

b) $\frac{3}{4} \div 1\frac{1}{4} = \frac{3}{4} \div \frac{5}{4} = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

<u>Thabo</u>

a) Thabo multiplied the two mixed numbers and didn't divide them. (The 2^{nd} number wasn't inverted)

b) $1\frac{1}{10} \div 2\frac{1}{4} = \frac{11}{10} \div \frac{9}{4} = \frac{11}{10} \times \frac{4}{9} = \frac{22}{45}$

<u>Romy</u>

a) Romy divided the whole numbers and the fractions separately

b) $10\frac{1}{8} \div 2\frac{1}{4} = \frac{81}{8} \div \frac{9}{4} = \frac{81}{8} \times \frac{4}{9} = \frac{9}{2} = 4\frac{1}{2}$

NOTE TO THE FACILITATOR FOR SESSION 3

(allow 90 minutes ($1\frac{1}{2}$ hours) for this session)

The purpose of this session is to get the Participants to firstly complete a magic square involving fractions and secondly design their own magic squares.

3.1: WHAT IS A MAGIC SQUARE?

20 minutes	*Explain what a magic square is* *Complete Activity 1 where the Participants first work out the Magic Total for the two magic squares and then uses the addition and subtraction of fractions to find the missing numbers in the magic square*

3.2: HOW TO DESIGN A 3 × 3 MAGIC SQUARE

20 minutes	*Explain how to make your own 3 × 3 magic square using a sequence of nine numbers* *Complete Activity 2 where the Participants first work out the common difference between the terms of two sequences, then use these sequences to create two 3 × 3 magic squares and finally use their own sequence of numbers to create a 3 × 3 magic square*

3.3: HOW TO DESIGN A 4 × 4 MAGIC SQUARE

20 minutes	*Explain how to make your own 4 × 4 magic square using a sequence of 16 numbers.* *Complete Activity 3 where the Participants first simplify two sequences of 16 numbers. They then create two 4 × 4 Magic Squares using the sequences of simplified numbers*

3.4: CONVERTING YOUR MAGIC SQUARE INTO AN ACTIVITY

30 minutes	*Explain how to turn a sequence of numbers into a 3 × 3 magic square and then remove sufficient numbers to turn the magic square into an addition and subtraction activity.* *Complete Activity 4 where firstly the Participants create their own sequence of 9 fractions, use this sequence to create a 3 × 3 Magic Square and then turn the Magic Square into an addition and subtraction activity, and finally discuss the usefulness of Magic Squares with their partners.*

SESSION 3: USING MAGIC SQUARES TO PRACTISE FRACTION CALCULATIONS

3.1. What is a magic square? (p 22)

The Class discusses what a Magic Square is.
The Participants work on Activity 1 with a partner, and this is marked.

A **3 by 3 Magic Square** is a square grid filled with nine distinct numbers. The sum of the numbers in ***each row*** is equal to the sum of the numbers in ***each column*** and is also equal to the sum of the numbers in the ***two main diagonals***

The following example is a magic square using fractions with identical denominators

1	$3\frac{1}{2}$	3
$4\frac{1}{2}$	$2\frac{1}{2}$	$\frac{1}{2}$
2	$1\frac{1}{2}$	4

ROWS	COLUMNS	MAIN DIAGONALS
$1 + 3\frac{1}{2} + 3 = 7\frac{1}{2}$	$1 + 4\frac{1}{2} + 2 = 7\frac{1}{2}$	$1 + 2\frac{1}{2} + 4 = 7\frac{1}{2}$
$4\frac{1}{2} + 2\frac{1}{2} + \frac{1}{2} = 7\frac{1}{2}$	$3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} = 7\frac{1}{2}$	$2 + 2\frac{1}{2} + 3 = 7\frac{1}{2}$
$2 + 1\frac{1}{2} + 4 = 7\frac{1}{2}$	$2 + 1\frac{1}{2} + 4 = 7\frac{1}{2}$	

$7\frac{1}{2}$ *is called the* ***constant*** or ***the magic total*** of *this magic square.*

Once you know what the constant or magic total is of a magic square, then it is possible to find the missing numbers.

ACTIVITY 1

- Find the magic total for each of these Magic Squares.
- Then use addition and subtraction to find the missing numbers.

SOLUTIONS

a) Identical denominators

$\frac{2}{5}$	$1\frac{2}{5}$	$1\frac{1}{5}$
$1\frac{4}{5}$	1	$\frac{1}{5}$
$\frac{4}{5}$	$\frac{3}{5}$	$1\frac{3}{5}$

Magic total = 3

b) One denominator is a multiple of the other

$\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{1}{2}$
$2\frac{1}{4}$	$1\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{3}{4}$	2

Magic total = $3\frac{3}{4}$

3.2 How to design a 3 by 3 Magic Square (p 23)

The Facilitator describes the steps to go through by writing on the board.
The Participants work on Activity 2 with their partner, and the work is marked.

STEP 1:
Decide on a sequence of ***nine*** numbers, each of which must differ from its neighbouring number by the same number. For example, use the sequence $\frac{1}{3},\ \frac{2}{3},\ 1,\ 1\frac{1}{3},\ 1\frac{2}{3},\ 2,\ 2\frac{1}{3},\ 2\frac{2}{3},\ 3$

STEP 2
Draw two grids like the following on squared paper

Table A **Table B**

STEP 3
Write the numbers in your sequence diagonally in Table A

Table A **Table B**

STEP 4
- Fill in all the numbers that are *inside* of the square in Table A into the same position in Table B
- Move each of the numbers that lie *outside* of the square in Table A to the *furthest* open block in its row or column in Table B.

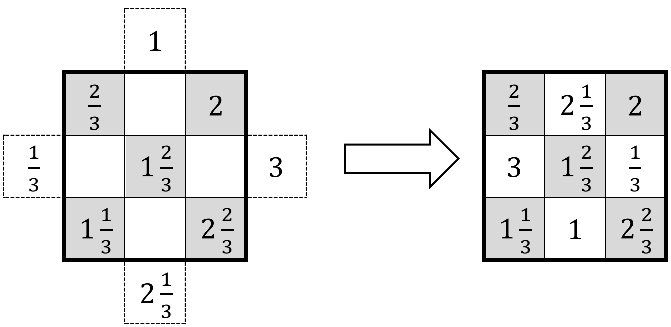


Table A **Table B**

In other words,
- move the 1 on top of the table to below the $1\frac{2}{3}$
- move the $2\frac{1}{3}$ from the bottom of the table to above the $1\frac{2}{3}$
- move the $\frac{1}{3}$ on the left of the table to the right of $1\frac{2}{3}$
- move the 3 on the right of the table to the left of the $1\frac{2}{3}$

STEP 5
Check that the sum of the numbers in every row, every column and each of the two main diagonals is 5.

ACTIVITY 2

1) For each of the following two sequences, determine what you add on to the first term to get the second term.

   a) $\frac{1}{2}$; $\frac{2}{2}$; $\frac{3}{2}$; $\frac{4}{2}$; $\frac{5}{2}$; $\frac{6}{2}$; $\frac{7}{2}$; $\frac{8}{2}$; $\frac{9}{2}$ ***common difference =*** $\mathbf{\frac{1}{2}}$

   b) $2\frac{1}{8}$; $2\frac{1}{4}$; $2\frac{3}{8}$; $2\frac{1}{2}$; $2\frac{5}{8}$; $2\frac{3}{4}$; $2\frac{7}{8}$; 3; $3\frac{1}{8}$ ***common difference =*** $\mathbf{\frac{1}{8}}$

2) Construct two 3 by 3 Magic Squares using the given sequences of numbers.
   Once you have completed the Magic Squares, check them by adding the numbers in the columns, rows and the two main diagonals.

a)

		$\frac{3}{2}$		
	$\frac{2}{2}$		$\frac{6}{2}$	
$\frac{1}{2}$		$\frac{5}{2}$		$\frac{9}{2}$
	$\frac{4}{2}$		$\frac{8}{2}$	
		$\frac{7}{2}$		

Table A

SOLUTION

$\frac{2}{2}$	$\frac{7}{2}$	$\frac{6}{2}$
$\frac{9}{2}$	$\frac{5}{2}$	$\frac{1}{2}$
$\frac{4}{2}$	$\frac{5}{2}$	$\frac{8}{2}$

Magic Number

$= 7\frac{1}{2}$

Table B

b)

Table A

$2\frac{1}{4}$	$2\frac{7}{8}$	$2\frac{3}{4}$
$3\frac{1}{8}$	$2\frac{5}{8}$	$2\frac{1}{8}$
$2\frac{1}{2}$	$2\frac{3}{8}$	3

Magic Number

$= 7\frac{7}{8}$

Table B

ACTIVITY 2 (continued)

3) Make up your own sequence of fractions and mixed numbers

……

Write the sequence of numbers in Table A. Use Table A to turn Table B into a Magic Square.

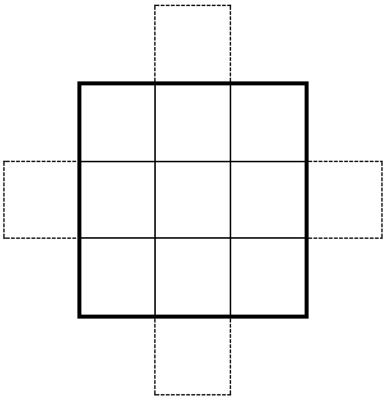


Table A

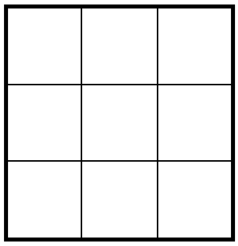
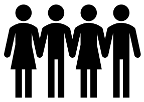


Table B

Once you have completed the magic square, check it by adding the numbers in the columns, rows and the two main diagonals.

Write the magic number here: ……………………………………………………….

3.3 <u>How to design a 4 by 4 Magic Square (p 26)</u>



The Facilitator describes the steps to go through by writing on the board.
The Participants work on Activity 3 with their partner, and the work is marked.

The method used for designing a **4 by 4 Magic Square** is completely different to the method used for designing a 3 by 3 Magic Square.

STEP 1: Decide on a sequence of ***sixteen*** numbers, each of which differs from its neighbouring number by the same number.

e.g. Start at 0 and add $\frac{1}{6}$th to each term to get

$0;\ \frac{1}{6};\ \frac{1}{3};\ \frac{1}{2};\ \frac{2}{3};\ \frac{5}{6};\ 1;\ 1\frac{1}{6};\ 1\frac{1}{3};\ 1\frac{1}{2};1\frac{2}{3};1\frac{5}{6};2;\ 2\frac{1}{6};2\frac{1}{3};\ 2\frac{1}{2}.$

STEP 2: Write the terms of the sequence in order in the table.

0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
$\frac{2}{3}$	$\frac{5}{6}$	1	$1\frac{1}{6}$
$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{2}{3}$	$1\frac{5}{6}$
2	$2\frac{1}{6}$	$2\frac{1}{3}$	$2\frac{1}{2}$

Is this a magic square?

STEP 3: Reverse the order of the numbers in each *main diagonal*

$2\frac{1}{2}$			2
	$1\frac{2}{3}$	$1\frac{1}{2}$	
	1	$\frac{5}{6}$	
$\frac{1}{2}$			0

STEP 4: Add the missing numbers back into their original positions

$2\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	2
$\frac{2}{3}$	$1\frac{2}{3}$	$1\frac{1}{2}$	$1\frac{1}{6}$
$1\frac{1}{3}$	1	$\frac{5}{6}$	$1\frac{5}{6}$
$\frac{1}{2}$	$2\frac{1}{6}$	$2\frac{1}{3}$	0

Is this a magic square?

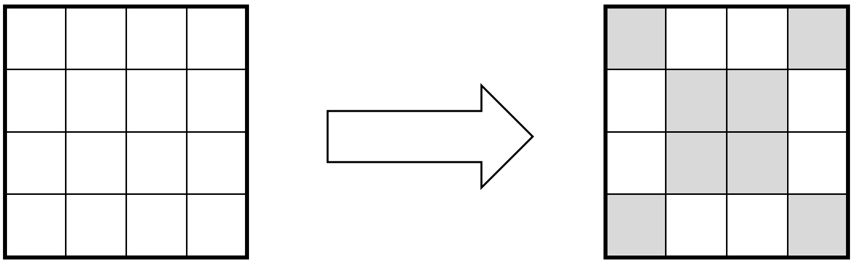
If so, what is its magic number? ………………………….

ACTIVITY 3

Use the following sequences of numbers to construct two 4 by 4 Magic Squares. Once you have completed each Magic Square, check it by adding the numbers in the columns, rows and the two main diagonals.

1) $\frac{1}{6}$; $\frac{2}{6}$; $\frac{3}{6}$; $\frac{4}{6}$; $\frac{5}{6}$; 1; $1\frac{1}{6}$; $1\frac{2}{6}$; $1\frac{3}{6}$; $1\frac{4}{6}$; $1\frac{5}{6}$; 2; $2\frac{1}{6}$; $2\frac{2}{6}$; $2\frac{3}{6}$; $2\frac{4}{6}$

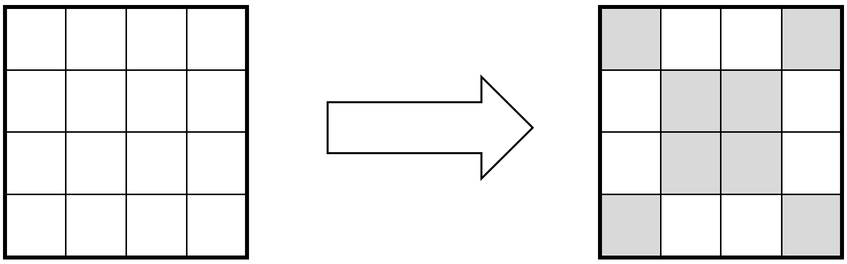
a)



Magic Number

= ………..

2) $\frac{1}{12}$; $\frac{2}{12}$; $\frac{3}{12}$; $\frac{4}{12}$; $\frac{5}{12}$; $\frac{6}{12}$; $\frac{7}{12}$; $\frac{8}{12}$; $\frac{9}{12}$; $\frac{10}{12}$; $\frac{11}{12}$; 1; $1\frac{1}{12}$; $1\frac{2}{12}$; $1\frac{3}{12}$; $1\frac{4}{12}$



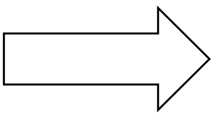
Magic Number

= …………

SOLUTION

a)

$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$
$\frac{5}{6}$	1	$1\frac{1}{6}$	$1\frac{2}{6}$
$1\frac{3}{6}$	$1\frac{4}{6}$	$1\frac{5}{6}$	2
$2\frac{1}{6}$	$2\frac{2}{6}$	$2\frac{3}{6}$	$2\frac{4}{6}$



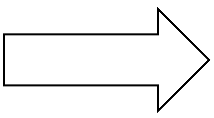
$2\frac{4}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$2\frac{1}{6}$
$\frac{5}{6}$	$1\frac{5}{6}$	$1\frac{4}{6}$	$1\frac{2}{6}$
$1\frac{3}{6}$	$1\frac{1}{6}$	1	2
$\frac{4}{6}$	$2\frac{2}{6}$	$2\frac{3}{6}$	$\frac{1}{6}$

Magic Number

= $5\frac{4}{6}$

b)

$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$
$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$
$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	1
$1\frac{1}{12}$	$1\frac{2}{12}$	$1\frac{3}{12}$	$1\frac{4}{12}$

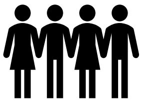


$1\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$1\frac{1}{12}$
$\frac{5}{12}$	$\frac{11}{12}$	$\frac{10}{12}$	$\frac{8}{12}$
$\frac{9}{12}$	$\frac{7}{12}$	$\frac{6}{12}$	1
$\frac{4}{12}$	$1\frac{2}{12}$	$1\frac{3}{12}$	$\frac{1}{12}$

Magic Number

= $2\frac{10}{12}$

3.4 <u>Converting your Magic Square into an Activity (p 28)</u>



The Facilitator describes the steps to go through by writing on the board.
The Participants work on Activity 4 question 1 on their own and the swap Magic Squares with a partner.
They then discuss with their partner the usefulness of Magic Squares when teaching fractions.

Magic Squares give the learners practice adding and subtracting whole numbers, fractions, decimals, and even negative numbers (integers).

In order to create a Magic Square Activity (like those in Activity 1) that you could give to the learners to solve, follow the following procedure:

STEP 1 – Create a Magic Square
Suppose you take the sequence 1; 3; 5; 7; 9; 11; 13; 15; 17

		5		
	3		11	
1		9		17
	7		15	
		13		

Table A

Becomes

3	13	11
17	9	1
7	5	15

Table B

STEP 2 – Copy either one complete row, one complete column, or one main diagonal into a blank 4 by 4 square

Example 1

3	13	11

Example 2

		11
	9	
7		

STEP 3 – Add in as few of the other numbers as possible

Example 1

3	13	11
	9	

Example 2

		11
	9	1
7		

Is it necessary to add in any other numbers or have you been given enough information to find the missing numbers?

ACTIVITY 4: Create a 3 by 3 Magic Square and turn it into an Activity

1) Create a 3 by 3 Magic Square by following the following steps:
   a) Create a 9-term sequence of fractions. (Don't use one we have used already.)

      ……………………………………………………………………………………………………

   b) Use the sequence to create a Magic Square on the squared paper below.
   c) Create your own Magic Square Activity by rewriting the Magic Square, removing it as many numbers as possible. Remember to leave in one column or one row or one main diagonal so that the person solving the Magic Square can work out the magic number before they find the missing terms.
   d) Give your Magic Square to one of your colleagues to complete. Make sure your cover your working out so they work out the Magic Square without clues.

2) Once you have finished this activity, discuss with your partner whether you think that you would use Magic Squares in your classroom. Do you think they are useful or not? Explain why.

   Generally magic squares are useful for practising the addition and subtraction of fractions in a non-standard form.
   They encourage problem solving and the methodical approach to a problem.
   You can adapt them to the level of your class by selecting a number sequence relevant to their level.
   You can use one magic square several times over by changing the starting numbers that you fill in.