education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Just-in-Time Training Workshop
Term 3, 2016

Facilitator Guide

Grade 4 – 7
Subject: Mathematics

Endorsed by:

A re Tokafatseng
Seemo sa Thuto
what I do matters

SAOU

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Programme

Session 1:	Reflection and Trackers	page 2	*20 mins*
Session 2:	Development in CAPS across three topics Geometric patterns, transformations, probability	page 4	20 mins
Session 3:	Probability	page 5	50 mins
	TEA BREAK		
Session 4:	Geometric patterns	page 8	70 mins
Session 5:	Transformations Tessellations, translation, reflection and rotation Enlargement and reductions	page 14	*70 mins*
Participants summary of key learnings from workshop			*10 mins*
PILO Evaluation of workshop			*10 mins*

Session 1: Reflection and Trackers

Refer to the reminder about the purpose and use of the Tracker on the next page. There may be some participants who need reminding.

Start the discussion about highlights with an encouraging example of your own involvment in schools e.g. curriculum coverage, more understanding of content, use of workshop materials, teachers who have given you positive feedback.

Session 2: Development of CAPS in three topics

Refer to the note about choosing the three topics for this workshop.

1. Weighting of topics *Don't spend more than 15 minutes on this!*
Let participants discuss weighting with a focus on the workshop topics with a partner for a few minutes. The percentages correspond with the hours allocated to topics in each grade's year plan.

They should notice that *Functions, Patterns and Algebra* becomes more important in Grade 7 and number becomes less important. This is because the number work learnt in FP and IP gets used more and more in functions and algebra. Patterns in IP are an essential step towards understanding algebra, so they are very important, even though they are only 10% of the year's work. Patterns help learners to do the transformations in shape and space, they build number concepts and logical thinking.

Time Allocation per Topic: Grade 4 – 7

Groups/partners can give you a little feedback. The important things to notice:

- **Geometric patterns**: 6 hours in Grades 4 – 6, 9 hours in Grade 7 (combined with number patterns), all done at similar times in the plan. This suggests that teachers of these grades can discuss and plan together across grades and help each other with resources. The same observation applies to **Transformations**.

- **Symmetry** in term 2 supports transformations in terms 3 and 4, particular reflections of shapes.

- Although **Probability** has only two hours in the fourth term, it should not be left out! Probability in IP prepares learners for Grade 7 work on probability, which is allocated 4½ hours.

Session 3: Probability

Check that teachers are able to verbally explain the word probability to their learners.

According to CAPS, Intermediate Phase learners only need to have some experiences of probability experiments. They need to be able to record the possible outcomes of an "event" and record the results of each event in a tally table.

Only in Grade 7, do learners begin to use the experiments to talk about "equally likely" events (50/50 chance, 1 in 2 chances etc).

Emphasise the need to use enough coins and die for groups of learners to do simple experiments themselves.

Session 4: Patterns

Activity 1

Use the discussion here to help teachers realise that patterns are present in so much of our daily lives, our sense of order, our sense of beauty, our planning and predicting.

Teachers also need to reflect on how work with mathematical patterns builds the learner's understanding of relationships between numbers and relationships between shapes. So much of mathematics is built on an understanding of patterns and mathematical structures.

Geometric patterns across the grades

The development of concepts:
If teachers don't raise these things in their discussion, here are some important points about geometric patterns in CAPS.

- The emphasis in **Foundation Phase** is on creating, copying and extending patterns that are made of physical objects or drawings. They extend patterns by drawing the next one or the next two in the sequence.

- At **Grade 4** level, learners begin to explain the rule or relationship that they see between the parts of a pattern. This is informal and it is enough for a learner to explain something like, "I added two on from the previous/last picture each time."

- Across **Grades 4 – 7**, different **equivalent representations** of patterns are developed – flow diagrams, number sentences, tables and formulae. In every grade, this must be supported by **verbal explanations** (they can be written in words). This is to build the learner's vocabulary and reasoning.

- In a geometric pattern, you can identify what shapes to add *and* where you need to put them in each picture. In number patterns, you only have the total number of shapes in each step of the pattern. The examples on pages 10 and 12 show this. The triangles on page 10 and the matchsticks on page 12 are arranged in a particular way to make the pattern. When we use the pattern to find a number pattern, then the positions of the objects are no longer important.

- Explain what CAPS means by: *sequences <u>not limited</u> to a constant difference or ratio*

  Patterns with a **constant ratio**:
  Each step of the pattern is mulitplied or divided by a constant number
  e.g. 3; 9; 27; 81; … (multiply by 3 each time) or 16; 8; 4; 2; 1; ½; ¼; … (divide by 2 each time)
  Patterns with a **constant difference**:
  Subtract or add a constant number from/to each step of the pattern.

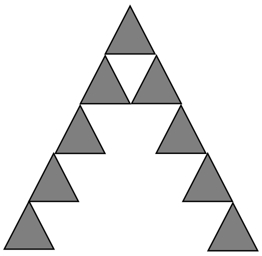
  e.g. 7; 10; 13; 16; … (add 3 each time) or 30; 25; 20; 15; … (subtract 5 each time)
  Other patterns (i.e. the ones that are not limited to a constant difference or a constant ratio) includes
  - patterns of squared numbers (e.g. 1; 4; 9; 16; …),
  - patterns of triangular numbers (1; 3; 6; 10; 15; …)
  - Fibonacci series (1; 1; 2; 3; 5; 8; 13; …)

Activity 2

Explain that this activity is for the teachers. It shows teachers an overview of the links between a geometric pattern and a flow diagra, tables, number sentence, function or graph that all represent the pattern. Take teachers through the process to the end where the graph is made. They should actually _do_ each part of the activity.

Answers

1. **Picture 5**



2. Teachers will find different ways to describe that you add two triangles of the same size as the others to the bottom of each diagonal line of triangles.
3. There may be some different explanations offered. Here is one: the 10^{th} shape will have two diagonal lines of triangles with 10 triangles in each. The top triangle of each line is the same triangle. So there are 20 – 1 triangles in the 10^{th} shape.
4. Two triangles are added each time (This only describes the change from one picture to the next; it does not give us a general rule that will work for any number of triangles).

5. Here is a reliable method to find the rule of the pattern.
Thank you to Lungile Mbambo, subject advisor.

We are adding 2 each time … this is like repeated addition, which is the same as mulitplying by 2. So we multiply the picture number by 2.
Then write number sentences for each picture:

1: 2 x 1 + ? = 1
2: 2 x 2 + ? = 3
3. 2 x 3 + ? = 5
4. 2 x 4 + ? = 7

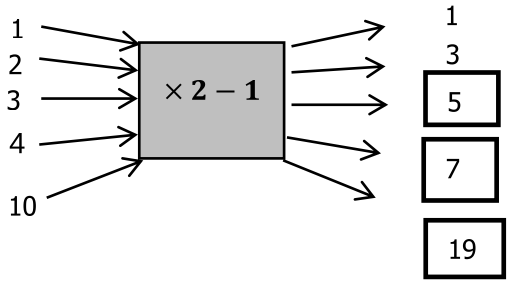
What number is needed to make each of the number sentences true? By simple trial and error, you can find that each number sentence needs –1.
2 x 1 – 1 = 1
2 x 2 – 1 = 3
2 x 3 – 1 = 5
2 x 4 – 1 = 7

2 x n – 1 = 2n – 1

In words, we can say take the position of the picture and double it and subtract 1.

The flow diagram and the table represent the same relationship as the sequence of triangles.

Position number Rule Number of triangles



Position	1	2	3	4	5	6	10	n
Number of triangles	1	3	5	**7**	**9**	**11**	**19**	**$2n – 1$**

6. For position 50, the number is 50 x 2 – 1 = 99?
7. Work backwards to find the answer, using inverse operations.
   So 37 – 1 = 36 and 36 ÷ 2 = 18. The 18th picture has 37 triangles.
8. $y = 2x - 1$

Position x	1	2	3	4	5	6	7	
Number y	1	3	5	7	9	11	13	

- Participants can plot more points on the graph if there is time.
- The points can be joined with a straight line because the rule that we found works for any of the numbers in between the ones we plotted … to infinity!

Activity 3: Matchstick patterns (answers)

a) Picture 5 has 5 squares.
b) 16 matches are needed to make 5 squares.
   100 squares: 3 x 100 + 1 = 301 matchsticks
   The first square uses 4 (3 + 1) matchsticks. Then each square after that uses another 3.
c) 454 matches: work backwards with inverse operations.
   454 – 1 = 453 and 453 ÷ 3 = 151.
   So the picture in position 151 will have 454 matchsticks!

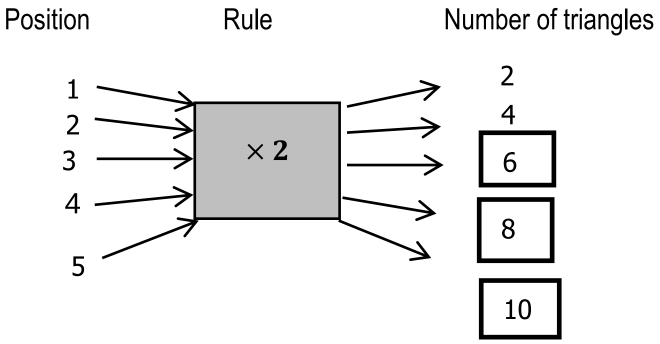
Activity 4

Explain that this activity gives teachers an understanding of the different types of geometric patterns and ways to represent them. Give participants enough time to do about 4 of the patterns and then get feedback.

Keep the discussion focused on verbal explanations and on question 2.

Answers:

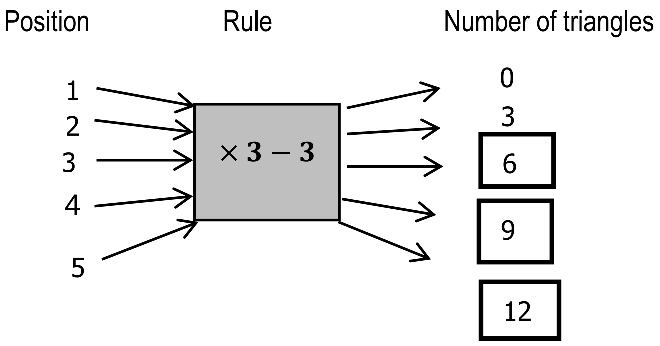
Pattern A:
We start with 2 triangles in a diamond shape and add 2 more triangles arranged in a diamond shape to each step to make a row of diamonds.



Position	1	2	3	4	5	6	7	*n*
Number of triangles	2	4	6	**8**	**10**	**12**	**14**	**2*n***

The n^{th} term will have 2 x *n* triangles.

Pattern B:
We start with no dots, then a 3-dot triangle. Then keep adding another 3-dot triangle to each step/position in the pattern. Build a tower of dots in this way.



Position	1	2	3	4	5	6	7	*n*
Number of triangles	0	3	6	**9**	**12**	**15**	**18**	**3*n* – 3**

The *n* th term will have 3 x *n* – 3 triangles.

Pattern C: Refer to the example on page 10. The number pattern is the same although the triangles are arranged differently.

Pattern D: A square number of dots each time, arranged in a square, starting from 1 dot, then 4 dots, then 9 dots. So the rows and the columns have the same number of dots in each picture.

Flow diagram: 1 → 1; 2 → 4; 3 → 9; 4 → 16 etc. The rule is n^2

Pattern E: Starting at 1 square, add one more square for each picture to make a row of squares.
Flow diagram: 1 → 1; 2 → 2; 3 → 3; 4 → 4 etc.

The rule is that the picture number is the same as the number of squares, so $n = n$.

Pattern F: Starting at 2 squares, add 2 more squares on top of the previous squares for each picture, making two towers of squares.
Flow diagram: 1 → 2; 2 → 4; 3 → 6; 4 → 8 etc.
The rule is that the number is double the picture number, so it is $2n$.

Pattern G: Starting at 2 dots in a tower (column), add 1 more row and 1 more column to each picture number.
Flow diagram: 1 → 2; 2 → 6; 3 → 12; 4 → 20 etc.
The rule is that the number of columns in each picture is the same as the number of the picture and the number of rows is one more than the number of the picture.
The rule is to multiply the picture number by one more than the picture number, so it is $n\ (n + 1)$.

Pattern H: Starting at 1 dot in the first picture, add 2 below it to make a triangle, then add 3 below for the next picture to make a bigger triangle, add 4 below in the next picture and continue in this way.

Flow diagram: 1 → 1; 2 → 3; 3 → 6; 4 → 10; 5 → 15 etc.
The rule for the nth term is that it will n rows of dots with (n) dots in the last row.
The total number of dots in this picture is (1 + 2 + 3 + … + n). This rule is not expected from Internediate Phase!

Pattern I: Starting at 2 triangles, add 4 triangles to make a hexagon, then keeping adding 4 triangles to create overlapping hexagons.
Flow diagram: 1 → 2; 2 → 6; 3 → 10; 4 → 14 etc.
The rule is to multiply the picture number by 4 and add 2, so it is $4n + 2$.

- For each pattern, learners from Grade 4 – 7 can draw the next picture and they can identify how to get from one picture/step to the next. All these grades can make a flow diagram, but they will struggle with the rule for some patterns.

- Grade 4 does not do the table of values.
- Only Grade 7 needs to learn to use the formula using x or n

Session 5: Transformations

Activity 1

Here is a suggested summary of CAPS.

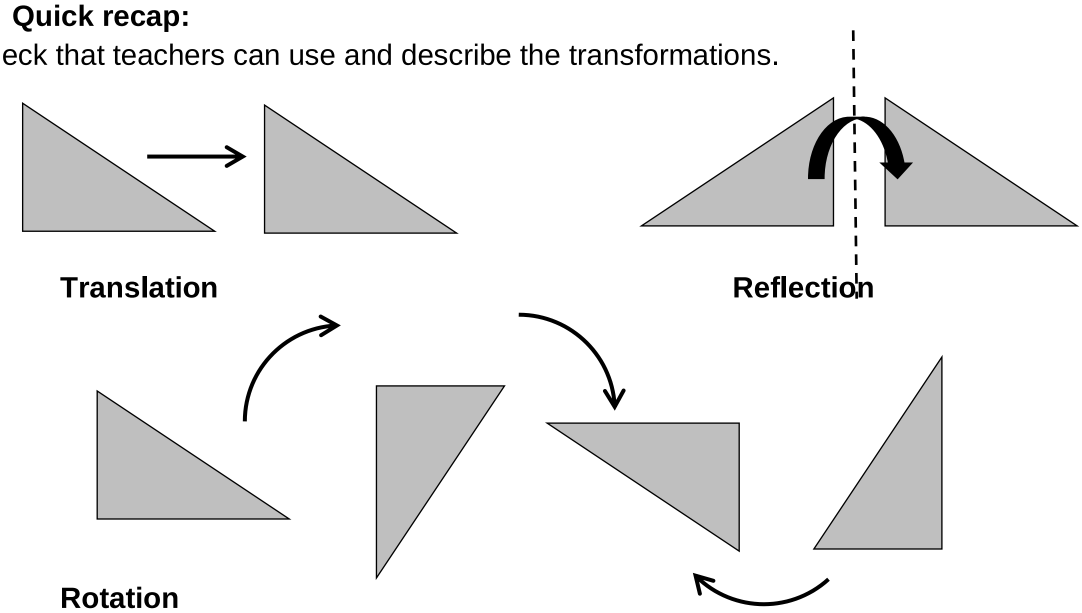
Grade 4	Grade 5	Grade 6	Grade 7
Build composite shapes; Include lines of symmetry.			Draw lines of symmetry
Tessellations: **Pack out** 2-D shapes	**Tessellations** and **transformations:** **Trace and move** shapes; translate, reflect and rotate	**Draw enlargements** and **reductions** of Δs and quadrilaterals compare shape and size	**Draw** all transformations on squared paper
Describe: Refer to lines, shapes, objects, lines of symmetry, (transformations of shapes) when describing patterns -- in nature; from modern everyday life; our cultural heritage			

- Grade 4 learners build shapes and look for lines of symmetry.
- This will support their transformation work in Grade 5 with transformations that do not change the size and shape of any shape, only the orientation.
- In Grade 6 learners work with enlargement and reduction, but only of triangles and quadrilaterals.
- In Grade 7, learners transform any shape with all types of transformations using squared paper.

Activity 2:

1. Quick recap:

Check that teachers can use and describe the transformations.

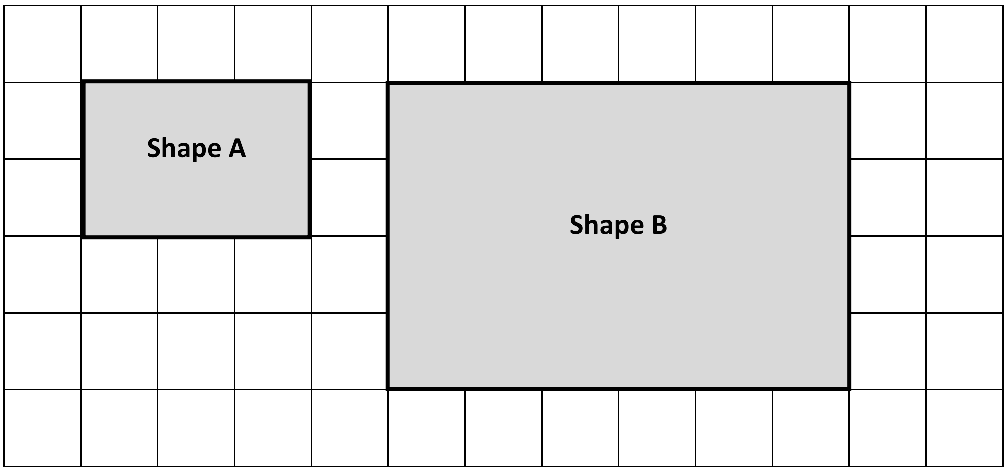


Check that teachers understand that “translate”, “reflect” and “rotate” are the actions related to translation, reflection and rotation.

It is helpful to think of translating as “sliding” a shape without any turning; reflecting as “flipping” or turning the shape over (so the bottom surface shows) and rotating and “turning”.

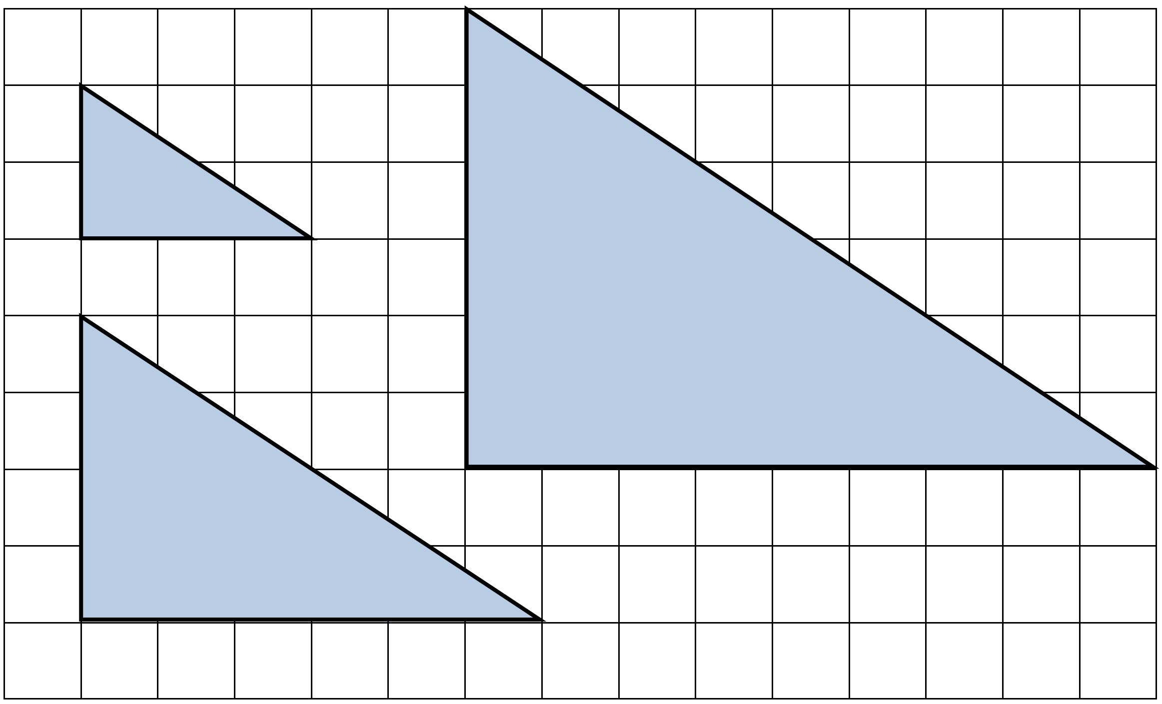
Exercise D

Use squared paper to draw a shape that is an enlargement of shape A.
Double the length of all the sides of shape A. Call your new shape B.



The area of shape A is 6 squares and the area of shape B is 24 squares, so shape B is 4 times bigger than shape A in area.
The perimeter of shape A is 10 units and the perimeter of shape B is 20 units, so it is double the perimeter of Shape A.

Exercise E



Exercise F

The new shapes must lengths of half of each side of the old shape. So the “arrow” shape has dimensions that fit halfway between dots on the dotty paper.