



2018 TRAINING WORKSHOP NO.2 MATHEMATICS



FOUNDATION PHASE



education

Department:

Education

PROVINCE OF KWAZULU-NATAL

Foundation phase

Just-in-Time Training Workshop 2018: No. 2

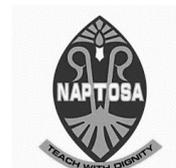
Participants' Handout

Mathematics



Jika iMfundo
what I do matters

Endorsed by:



Jika iMfundo
Foundation Phase JIT 2 of 2018
Mathematics
Workshop handout for Participants

In this workshop participants will find out more about:

- Problem solving using the Jika iMfundo Toolkit
- How to teach problem solving in mathematics

Participants will work in groups on all of the activity questions. Time guidelines are given and the facilitator will interact with the participants while they work. Participants will also be able to share key ideas together with the large group

Materials:

Participants will need the following materials

1. Term 3 lesson plans and trackers for Grades 1 to 3. Extracts that are relevant to each grade are included in the participants' hand-out.
2. Counters (about 50 counters per group of 5 participants)
3. Paper for participants to work on

SUGGESTED WORKSHOP PLAN		
Time	Session/Topic	Duration
8H00 - 8H30	<ul style="list-style-type: none"> ○ Arrival/ Register ○ Distribution of workshop material ○ Introduction to the workshop 	30 mins
8H30 - 13H30	Problem solving	
	A. The need to teach problem solving	20 mins
	B. What are problems? Types of problems and broad approaches to teaching problem solving	30 mins
	C. Problem solving in the CAPS for FP	40 mins
	D. Types of knowledge required to solve problems in Maths	20 mins
	E. Mathematical proficiency (20 mins
	F. Developing basic operations through problem solving	40 mins
	G. Problem solving using different operation strategies in the Jika iMfundo lesson plans	70 mins
	H. Reflective practice in the context of learners' problem solving	30 mins
		5 hours
13:30	Closure	
13:40	Lunch	

A. The need to teach problem solving

[You will spend about 20 minutes on Section A]

Discussion and input by facilitator

Activity 1: Think about your own problem solving (10 minutes)

Getting yourself to a training workshop in the course of a busy week presents problems that you need to solve. What problems arose for you and how did you resolve them in order to be here at today's workshop?

Discussion of responses and input by facilitator

Problem solving trains us for real life. Our mathematics training can be seen as equipping us for everyday situations and some of the problems we are confronted with in life. We need to approach a problem systematically. **Consider the following steps which could guide you towards successful problem-solving.**

Step 1: Read the problem carefully and ensure that you understand what the problem is about. Restating the problem in your own words is a good exercise, which will make it clear to you whether or not you have understood the meaning of the problem. It is often a good idea to try and sketch a diagram that assists you to illustrate what is required by the problem.

Step 2: Once you have understood what the problem is asking, you have to think of your strategy for solving the problem. Think about whether you have all the information that you need in order to solve the problem? Have you solved other similar problems which can guide your solution to the current problem? And, can the problem be broken up into smaller parts if it seems too big to solve all at once?

Step 3: Here you go about implementing your problem-solving strategy to get to the actual solution to the problem. It is important that you realise the difference between devising a strategy to solve a problem and the actual solution to the problem. Both are important activities. It will become clear to you if you need to change your strategy or find a new one, or if your original strategy was adequate.

Step 4: Once you have solved the problem, a final "logic check" of your solution is never a waste of time. Careless errors can slip into your working (though your strategy may be correct) and lead you to an answer which is not correct. Re-read your work just to be sure that it makes sense and presents a valid, satisfactory solution to the problem. This step of verification may seem like a waste of time, but will often prove its usefulness when on verification; you make small changes and improvements to your answers.

Activity 2: Summarise the key steps in solving a problem (10 min)

How would you summarise each of steps 1 to 4 above in one sentence or phrase?

Step 1:

Step 2:

Step 3:

Step 4:

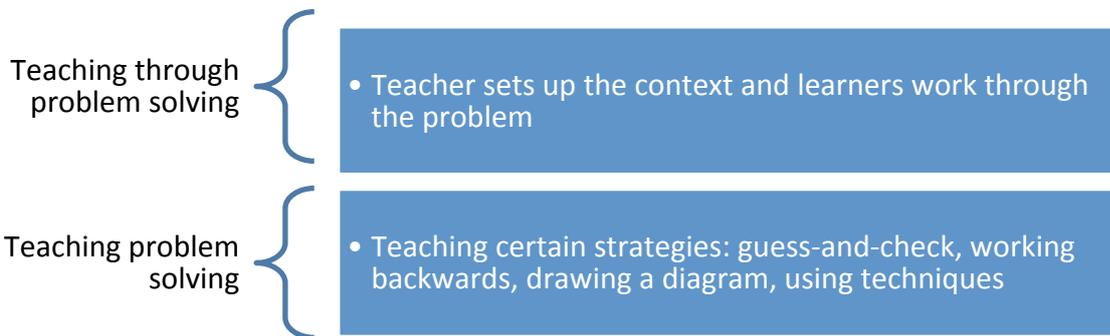
B. What are problems? Types of problems and broad approaches to teaching problem solving
 [You will spend about 30 mins on Section B]

What is a problem? Discussion and input by facilitator

Distinction between teaching problem solving and teaching through problem solving - discussion and input by facilitator

Figure 1 below summarises the key features of teaching problem solving and teaching through problem solving

Figure 1: Teaching through problem solving and teaching problem solving



Types of problems

Problems are categorised in two different ways.

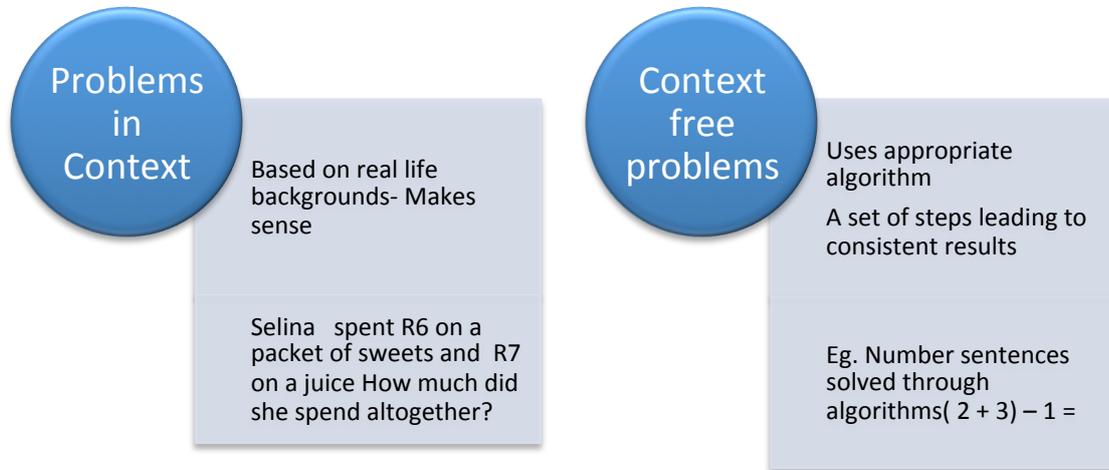
Firstly, as **routine and non-routine problems** as seen in Table 1 below

Table 1: Difference between routine and non-routine problems

Routine Problems	Non Routine problems
<ul style="list-style-type: none"> • Use known or prescribed procedures in obtaining solutions • Use traditional worded problems to enable students to use standard algorithms • The problem to be solved is similar to one that has been done before. Algorithms can be seen as: rules for calculating computational procedures logical step by step procedures 	<ul style="list-style-type: none"> • Use more than one strategy or solution • No algorithms exist • Solved through the use of heuristics and not algorithms <p>Heuristics are:</p> <ul style="list-style-type: none"> • Procedures or strategies that do not guarantee a solution • But provides a possible way to discover a solution <p>Eg. Building a model or drawing</p>

Secondly, problem types are categorized as “**problems in context**” and “**context - free calculations (CAPS)**”. Figure 2 below represents the distinction.

Figure 2: Problems in context and context free problems



Activity 3: Compare routine and non-routine problems (10 minutes)

Look at the examples of a non-routine and a routine problem and then discuss with the person sitting next to you the features of non-routine problems. You may compare this to the routine problem. Is it a straightforward question? Is there just one answer? Is there any one specific method to get to the answer?

Non routine problem

If hot dog rolls come in bags of 8 and viennas are sold in packs of 12, what is the least number of each pack that a person must buy in order to have 1 vienna for every roll? List other numbers that will ensure that a person has 1 vienna for every roll without any remainders.

Routine problem

Thembi bought a pack of 10 viennas and a dozen rolls. How many more rolls are there than viennas?

Discussion of responses to Activity 3

C. Problem solving in the Foundation Phase according to CAPS

[You will spend about 40 minutes on section C]

In this section, participants will develop their understanding of problems in context relevant to Foundation phase teaching as stated in CAPS. Teachers will refer to Table 2 below to answer questions to demonstrate their understanding of the policy requirements related to “**Problems in context**”

Table 2: Solving Problems in context

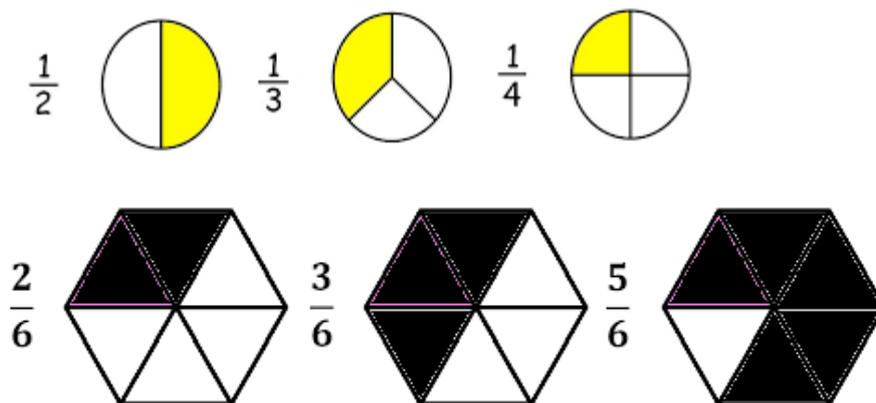
(Adapted from the Foundation phase overview (CAPS, 2011: 20) with numbering as in the original.)

Solving problems in context	Grade 1	Grade 2	Grade 3
1.6. Problem solving techniques	Use the following techniques when solving problems and explain solutions to problems: <ul style="list-style-type: none"> • Use concrete apparatus e.g. counters • Draw pictures to draw the story sum • building up and breaking down number • doubling and halving • number lines supported by concrete apparatus 	Use the following techniques when solving problems and explain solutions to problems: <ul style="list-style-type: none"> • drawings or concrete apparatus e.g. counters • building up and breaking down number • doubling and halving • number lines 	Use the following techniques when solving problems and explain solutions to problems: <ul style="list-style-type: none"> • building up and breaking down number • doubling and halving • number lines • rounding off in tens
1.7 Addition and subtraction	Solve word problems (story sums) in context and explains own solutions to problems involving addition and subtraction with answers up to 20	Solve word problems (story sums) in context and explains own solutions to problems involving addition and subtraction with answers up to 99	Solve word problems (story sums) in context and explains own solutions to problems involving addition and subtraction with answers up to 999
1.8 Repeated addition leading to multiplication	Solve word problems (story sums) in context and explains own solutions to problems involving repeated addition with answers up to 20	Solve word problems (story sums) in context and explains own solutions to problems involving repeated addition with answers up to 50	Solve word problems (story sums) in context and explains own solutions to problems involving repeated addition with answers up to 100
1.9 Grouping and sharing leading to division	Solve and explain solutions to practical problems involving equal sharing and grouping with whole numbers up to 20 and with answers that may include remainders	Solve and explain solutions to practical problems involving equal sharing and grouping with whole numbers up to 50 and with answers that may include remainders	Solve and explain solutions to practical problems involving equal sharing and grouping with whole numbers up to 100 and with answers that may include remainders
1.10 Sharing leading to fractions		Solve and explain solutions to practical problems involving equal sharing leading to solutions that include unitary fractions	Solve and explain solutions to practical problems involving equal sharing leading to solutions that include unitary and non-unitary fractions

Activity 4: Interpreting the extract from CAPS. (20 mins)

This activity has 12 questions, based on Table 2

1. What is meant by “problem solving techniques” as stated in 1.6 in Table 2? Give some examples of problem solving techniques.
2. What knowledge and skills will learners need to “explain solutions to problems”?
3. Compare the problem solving techniques used in Grade 1, 2 and 3
4. Explain how you understand the outcome: “Explain solutions in context involving addition and subtraction”. What does the word “involving” mean?.
5. What does it mean to “find a fraction of a whole”?
6. Explain how the problems related to “Sharing leading to fractions” differ in each grade.
7. The curriculum uses the terminology “unitary” and “non-unitary”. How do these fractions differ? Give some numeric examples of “unitary” and “non-unitary” fractions.
8. How does sharing leading to fractions differ across the grades?
9. What is meant by the phrase “problems that involve equal sharing”?
10. Why is equal sharing essential in the context of fraction concept development?
11. Which of the following are examples of “unitary” fractions and which are examples of “non-unitary” fractions? What makes them different?



1. Why do you think it is important for teachers to have a good understanding of Table 2?

Discussion of responses

The next sections will focus on the different types of knowledge and the five strands of mathematical proficiency. These two aspects are related to mathematical thinking which is the cornerstone of problem solving as stated in CAPS.

Discussion and input by facilitator

D. Types of knowledge required to develop problem solving

[You will spend about 20 minutes on Section D]

To help learners to think mathematically, it is imperative for the teacher to have a thorough understanding of the different kinds of knowledge, namely physical, social and conceptual (See Figure 3) and the five strands of Mathematical proficiency, namely understanding, applying, reasoning, engaging and computing which will be returned to later.

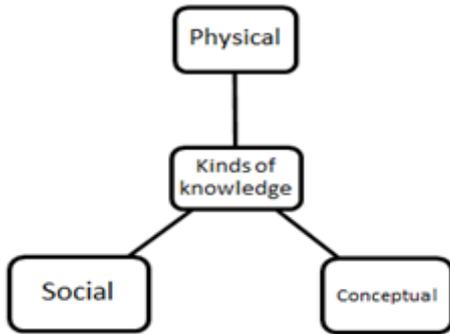


Figure 3: Types of knowledge

- Physical knowledge is derived through touching, using and playing with concrete materials.
- Social knowledge refers to knowledge that needs to be told to people and remembered by them as social knowledge
- Conceptual knowledge is knowledge that is constructed internally by each individual by themselves

Table 3 highlights some of the key characteristics of each kind of knowledge.

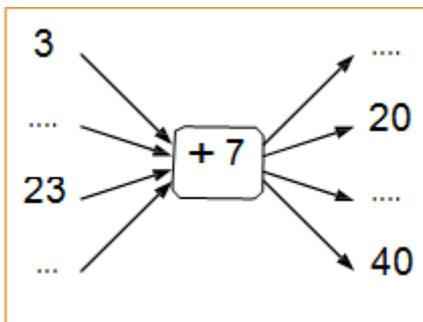
Table 3: Characteristics of three types of knowledge

Physical knowledge	Social knowledge	Conceptual knowledge
<ul style="list-style-type: none"> • Acquired through interaction with the physical world e.g. through touching, observing and handling objects • Learners need concrete experiences to develop physical knowledge • Problem solving- When learners draw, they use physical understanding of the problem • Implications- Mathematics classroom must contain lots of concrete apparatus 	<ul style="list-style-type: none"> • Knowledge that needs to be told by people and remembered • Conventions and rules are examples • The words add, subtract, minus and the symbols used to represent them • Teacher has to distinguish between knowledge that must be told and knowledge that must be constructed. Knowledge that can be developed through constructivism should not be told 	<ul style="list-style-type: none"> • Internal knowledge, constructed by each individual themselves • Teacher creates activities and situations, encourages learner to reflect on what they are doing • Children must express their thoughts in words, actions and methods • Cannot be taught through direct instruction

Activity 5: Planning activities that develop different kinds of mathematical knowledge (15 minutes)

This activity has 4 questions to complete. You should spend about 5 minutes on each question

- Participants are divided into 5 groups. Each group is allocated a topic (Numbers operations and relationship, patterns, space and shape, measurement and data handling). The groups plan different activities that include different type of knowledge, i.e. physical, social and conceptual, to teach any concept related to the topic that is assigned to them**
- “Completing tens (or hundreds)” is an important skill that learners need to develop in order to break down and build up numbers with confidence. To support learners in developing this skill a teacher might ask her class to complete a number of flow diagrams such as the one below. The teacher wants the learners to observe the pattern eg. “Adding a seven to a number ending in 3 completes the ten”. **Please complete questions A to D in the box below.**



Complete the calculations in the spider diagram. Then answer the questions that follow:

- After completing the answers, describe the pattern you identified.
- Can you apply this pattern to another example?
- How do you think this activity can lead to conceptual development?
- “Telling learners the rule” makes it easier for learners to learn than asking learners to identify patterns and apply patterns. To what extent do you agree with this statement?

- Study the two scenarios on counting below. Then identify the kinds of knowledge evident in each classroom. Provide a reason for your answer. Also consider the merits/ weaknesses of these strategies.**

A. Ms Ncube taught her grade one learners the number names. She then asked the learners as a class to count orally (chanting number names) from 1 to 100. Thabo struggled at 29, and then he would say twenty ten, twenty eleven, etc. But as he listened to other learners count, he was then able to count properly.

B. Ms Dlamini asked her grade one learners to count the number of objects in a pile by touching each object as they counted. She then asked the learners to make groups of tens, some counted in fives.

- Study the scenario on patterns below and answer the questions that follow:**

The learners in Ms Higg’s class are determining the values of $18 - 9$; $15 - 9$; and $13 - 9$. They were given several examples to practice with their concrete resources. Ms Higgs used two strategies:

Method 1: Used numberline to teach technique of “rounding off”. E.g. $18 - 9 = \dots$. Round off 9 by adding 1 to get 10. So if they added 1 to 9 they add 1 to 18 to get 19 (Because what we do to the left, we must do on the right) $19 - 10 = 9$.

Method 2: Taught them a rule: A quicker way is to remember the easy rule: “When we subtract 9 from 18 we add the 1 and the 8 to get 9 – this is our answer. Similarly when we subtract 9 from 15 we add the 1 and the 5 to get 6 – this is our answer.

5. i) Identify the type of knowledge in above methods. Provide a reason for your answer.
 ii) Which method do you think it most effective and why? What could be a limitation of this method?

Discussion of responses to questions

Next, let's look at what it means to be mathematically proficient.

E. Mathematical Proficiency

[You will spend 20 minutes on section E]

What does it mean to do mathematics?

Input and discussion by the facilitator

Learners are described as mathematically proficient if they can demonstrate all five of the competencies as seen in the Figure 4 below:

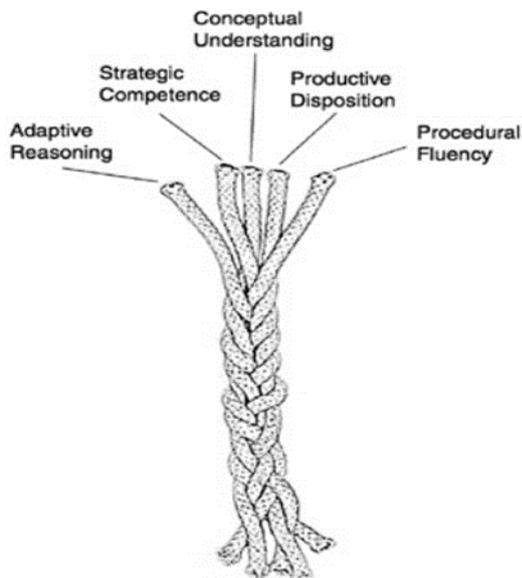


Figure 4: Mathematics Proficiency

- Conceptual understanding—an understanding of concepts, operations and relations. Learners are able to comprehend connections and see similarities between interrelated facts.
- Procedural fluency—flexibility, accuracy and efficiency in implementing appropriate procedures. This includes efficiency and accuracy in basic computations.
- Strategic competence—the ability to formulate, represent and solve mathematical problems. This is similar to problem solving. Strategic competence is mutually supportive with conceptual understanding and procedural fluency.
- Adaptive reasoning—the capacity to think logically about concepts and conceptual relationships. Reasoning is needed to navigate through the various procedures, facts and concepts to arrive at solutions.
- Productive disposition—positive perceptions about mathematics. This develops as students gain more mathematical understanding and become capable of learning and doing mathematics.

Activity 6. Mathematical proficiency. This activity has 2 questions. (10 minutes)

1. Why it is important for teachers to have a good knowledge of the five strands of mathematical proficiency?
2. Answer the questions in the box beside Lerato's response below.

Lerato (Grade 2) is solving the problem: 4 learners are paid R72 altogether. If they share the money equally between themselves, how much will each person get?

Lerato's Problem solving:
Grade 2

1. What strand of mathematical competency is reflected in Lerato's response? Provide a reason
2. Analyse the different steps used in the process and discuss what you think Lerato was thinking during the process.

Discussion of responses

F. Developing basic operations through problem solving

[You will spend 40 minutes on section F]

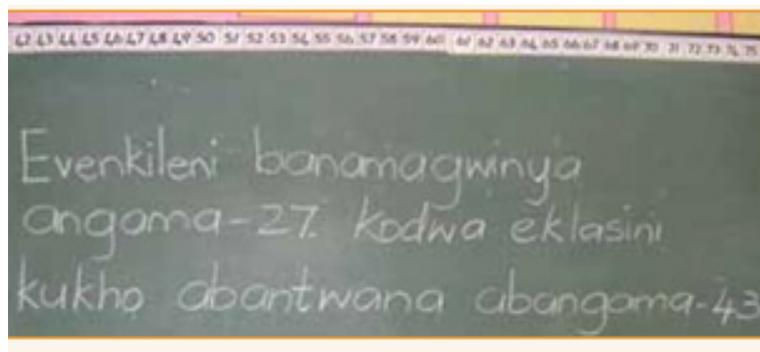
In this section, participants will find out more about how basic operations such as addition and subtraction can be developed through problem solving

Activity 7. Analyse problems and learner responses involving number operations (20 minutes for activity)

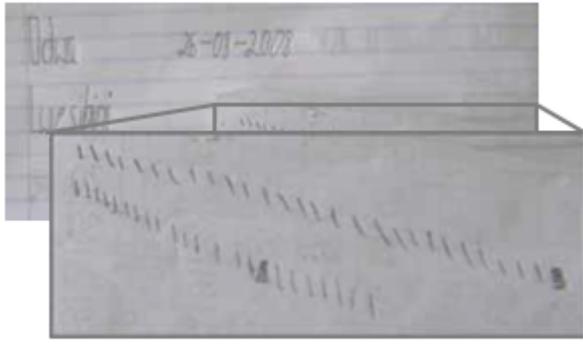
Answer 4 questions in this activity

1. Study the responses of two learners to the problem about amagwinyas. Discuss in your group how the problem was used to teach number operations. Also compare the two approaches used by the learners.

The grade 2 learners were given the problem: The tuck-shop has made 27 amagwinya (vetkoek). There are 43 learners in the class. Are there enough amagwinya for each child to get one? After some class discussion it was agreed that there were not enough amagwinyas and the teacher asked the learners to determine how many more were needed.



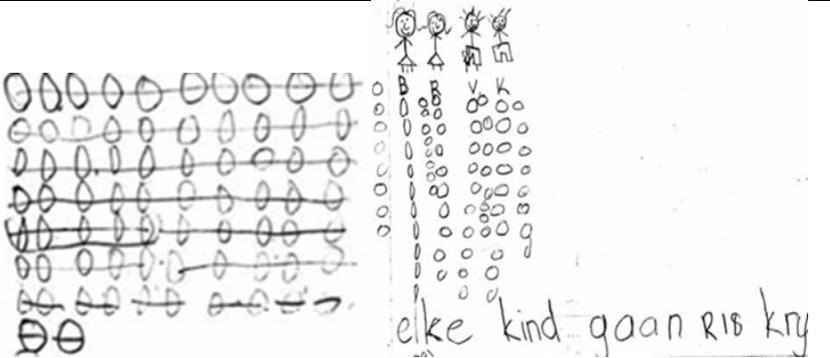
The following were the responses from two learners, namely Odwa and Andrew

<p>Odwa solved the problem by first drawing 27 stripes to represent the 27 learners in the class and then he drew a large number of extra stripes. He counted on from 27: 28, 29, 30 ... 43 and highlighted the 43rd stripe. Finally he counted how many extra amagwinya (stripes) were needed and concluded that 16 more amagwinya needed to be made. We can summarise Odwa's method as follows: $27 + 16 = 43$</p>	
<p>Andrew solved the problem by first counting out 43 counters. Next he counted out 27 from the 43 – as if he was giving amagwinya to those who he could give to. Finally he counted the remaining counters and established that he still needed 16 amagwinya for the remaining learners. We can summarise Andrew's method as follows: $43 - 27 = 16$.</p>	
<p>The teacher then went on to explain the different approaches used by both Odwa and Andrew. Then only did she introduce the notations and what it meant.</p>	

Discussion of response to question 1 (Activity on Amagwinyas)

2. The following problem was solved by two grade three learners (Mandy and Sally) who used different approaches as illustrated below. Analyse their approaches and answer the questions that follow.

Brenda, Roleen, Vuyani and Casper help Mr Kumalo in his vegetable garden on a Saturday morning. The children all work just as long as each other. Mr Khumalo gives them R72 altogether. How must the children share this amount evenly among them?

 <p>elke kind gaan R18 kry</p>	<p><i>Mandy's work</i></p>
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	Sally's work
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- 2.1. Interpret what the learners were thinking as they solved the problem.
- 2.2. Why is it important for children to use illustrations when solving problems?
- 2.3 Why do you think it is important for teachers to study learner's illustrations in problem solving?

Discussion of responses to question 2

3. In this activity, you will notice that different approaches may be used to solve a problem. Below are two sets of problems; set A and set B.

- 3.1 Work through the problems in SET A on your own.
- 3.2 Compare your approach with the approach used by the person sitting next to you. You will realize that in all likelihood, the approaches used to get the solution differ. Try to identify the key differences in the various solutions. How might learners approach them?
- 3.3 Work through the problems in SET B on your own.
- 3.4 Compare your approach with the approach used by the person sitting next to you. You will realize that in all likelihood, the approaches used to get the solution differ. Try to identify the key differences in the various solutions. How would learners approach them?
- 3.5 Why do you think it is important for learners to be allowed to solve these problems in different ways?

Problem set A	Problem set B
1. Arnold has 17 marbles and loses 11. How many marbles does he have left?	1. Anna has R24. A packet of chips costs R3. How many packets of chips can he buy?
2. Arnold has 17 marbles and Brendon has 11 marbles. How many more marbles does Arnold have?	2. Mother has 24 biscuits. She shares these equally among 3 learners. How many biscuits will each child get?
3. Arnold has 17 marbles and Brendon has 11 marbles. How many more marbles should Brendon get to have just as many marbles as Arnold?	

Discussion of responses question 3

4. Study the scenario below and discuss the possible reasons for Nozipho's errors.

Ms Dlamini, a grade two teacher, teaches each number operation separately. She felt that when she asked learners to solve problems that had different operations, it confused learners. The class then had to solve the following problem.

Bongi has 16 marbles. If he wins another 25, how many marbles will he have?

Nozipho did not hesitate, she wrote $16 + 25 = 41$. She then looked at her neighbour's work and seeing that he had written 42 she changed her answer to 42. Ignoring this for a moment, we could at this stage (based on her answer) be forgiven for thinking that Nozipho understands what she is doing. Immediately after responding to the first problem, Nozipho was asked to solve the problem:

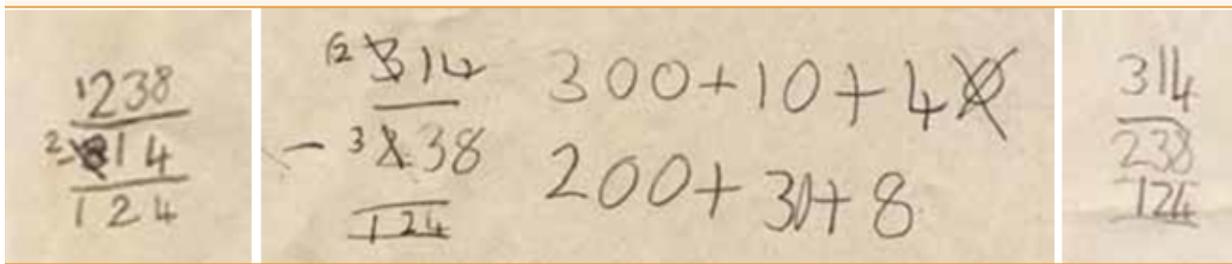
There are 28 apples. If we put 3 in a packet, how many packets can we fill?

Once again Nozipho hardly hesitated – she wrote: $28 + 3 = 31$.

Discussion of responses to question 4

5. Below are examples of learners' responses to the calculation problem: $314 - 238 =$

Identify the errors in the three examples and discuss the possible reasons for the learners' errors in the illustrations below



Discussion of Response to question 5

Through completing section F, you would have learnt that:

G. Problem solving using different operation strategies in the Jika iMfundo lesson plans

[You will spend 70 minutes on section G]

In this section, participants examine examples of problem solving exercises from the Jika iMfundo Term 3 lesson plans, answering questions or following instructions set on each. Through this activity, participants will familiarize themselves with the range of problem solving strategies and approaches taught to the learners in each grade. There will be discussion of participant's responses at appropriate places as they work through the activities.

Activity 8

GRADE 1: There are 5 questions to be answered in 20 minutes

Participants work in pairs...

1. Solve the problem in lesson 30. Then identify the different types of knowledge required by the learners to be able to solve this problem

Lesson 30: Using repeated addition: There are 3 friends. They have 5 sweets each. How many

sweets do they have altogether? Draw counters and write a number sentence

2. Look at the extracts from lessons 31a and 31b

Together with a partner, role play these lessons, using the prepared drawings of bicycles and sweets. Then answer the questions below:

- 2.1. What do you notice about the questions in 31a and 31b
- 2.2. What is the exercise designed to teach the learners?
- 2.3. The teacher could have told the learners that $3 \times 2 = 6$, $2 \times 5 = 10$ and drilled them on this. What do you think are the strengths/weakness of that approach compared with the one in the lesson plans?
- 2.4. How could you support a struggling learner?

Lesson 31a

Show a picture of 3 bicycles each with 2 wheels.



- What do you see?
- How many wheels on each bicycle?
- How many groups of 2 do you see?
- Let's count in twos.
- How many wheels altogether?
- How can I show that as a number sentence?
- How many times 2 do I have

Lesson 31b

Show a picture of 5 packets with 2 sweets in each



- What do you see?
- How many sweets in each packet?
- How many groups of 2 do you see?
- Let's count in twos.
- How many sweets altogether?
- How can I show that as a number sentence?
- How many times two do I have

3. Solve the following examples that show grouping and sharing with remainders. Then state how the example in 35a differs from the example in 35b. Why is it important to vary the examples for learners?

Lesson 35a



How many groups of 4 can you make?



How many groups of 3 can you make?



How many groups of 4 can you make?

Lesson 35b Share the balls between the given numbers of friends:



Share 11 balls between 2 friends. Each one gets balls.



Share 15 balls between 4 friends. Each one gets \square balls.

4. Complete the following problems by using the strategy of drawing. Explain why you think drawing is a recommended strategy in Grade one, but not in higher grades.

4.1 Thompho has 15 flowers. She puts three flowers into each bunch. How many bunches can she make?

4.2 Ntombela has 9 pencil crayons. She packs five crayons into a box. How many boxes will she need?

4.3 Draw to solve the following problem.



Share the pencils between 7 learners. How many pencils will each child get?

4.4 

Share the books between 3 learners. How many books will each child get?

5. Use counters to solve the problems below. Think about when counters will be used and when drawings will be used.

5.1 Cleo has 20 cookies that must be packed into boxes. 5 cookies go in each box. How many boxes will she need?

5.2 Tino has 11 marbles that he puts into groups. He puts 3 marbles in each group. How many groups does he have?

GRADE 2: There are 3 broad questions with sub questions to be answered in 25 minutes

Participants work in pairs and practice grade two exercises taken from Jika iMfundo. These exercises focus on strategies using the techniques of doubling, near doubles and breaking down/building up to solve problems. Responses will be discussed at appropriate places during the activity.

1. Lesson 11 (a, b and c)

Participants will role these exercises with a partner and then answer the questions that follow. The lesson plan uses unifix cubes, but participants can use counters or flard cards.

a. Using technique of doubling

Give each group of learners 75 Unifix blocks.

Ask various learners to make towers of ten blocks each in front of the class.

They are going to take turns to show doubles, using the blocks.

Put down 24 and 24 blocks in groups of 10 and ones. Ask the class: What is the sum?

Is there another way to say it?

Repeat allowing different learners to participate in the demonstration.

Show 37 and 37 blocks in groups of 10 and ones on the desk. Ask: What is the sum
Is there another way to say it?
Etc. work through more examples of doubles.

b. Using the technique of near doubles

Use unifixes cubes to solve:

$$28 + 29 = 57 \text{ (Double } 28 + 1 = 56 + 1 = 57.)$$

$$32 + 33 = 65 \text{ (Double } 32 + 1 = 64 + 1 = 65)$$

Practice this example. You can draw representations of base tens

$$35 + 34 = \quad 45 + 46 = \quad 43 + 45 =$$

c. Using the techniques of breaking down and building up

Give each group of learners a set of base ten blocks or counters. (If you don't have base ten blocks allow learners to work with flard cards.)

Use base ten blocks/ counters/ flard cards to show how to add the following:

Addition: $23 + 41 =$ and $45 + 27 =$

Subtraction: $55 - 31 =$ and $61 - 48 =$

Calculate using regrouping and exchange strategy. Do not draw pictures show numerical working?

$$54 - 16 = \quad 29 + 37 = \quad 48 - 19 =$$

Questions related to lesson 11a, b, and c

5.3. Why do you think it is important for learners to know the facts of doubling of single digit numbers from memory?

5.4. Why is doubling 37 more difficult than doubling 34? What knowledge and skills do learners need to double 37?

5.5. Explain your understanding of “using the technique of near doubles to solve calculations”.

5.6. Explain how the following sets of calculation in lesson 11c differ:

- $23 + 41$ and $45 + 27$
- $55 - 31$ and $61 - 48$

2. Lesson 24: Solving problems through tables and grids

Participants will practice these examples and talk about: the value of using tables and grids and how it contributes to learners' understanding of multiplication

a) Heila sells hotdogs at R4 each. Make a table to help her find the amount for large orders.

Number of hotdogs	1	2	3	4	5	6	7	8	9	10
Cost in rands	4	8	12	16	20	24	28	32	36	40

b) Show how to read the table to find out the costs of large orders. For example,

- *If she sells 4 hotdogs, she gets R__?*
- *If she sells 7 hotdogs, she gets R__?*

- If she sells 9 hotdogs, she gets R__? (
- c) Peter babysits. He charges R5 per hour for babysitting. Complete this table for him.

Number of hours	1	2	3	4	5	6	7	8	9	10
Cost in rands										

d) Use a grid to solve the following problem.

27 learners and 1 teacher go on a school trip to a nature reserve. The school pays R1, 20 per person to enter the nature reserve. How much money is paid in total?

- What is the key word? What is the question? What are the numbers?
- Draw a picture to show the answer.
- 27 learners + 1 teacher = 28 people (each pay R1,20).
- Draw a table with 7 columns and 4 rows because it will have 28 blocks.
- To find the total amount paid you have to add all the R1, 20 amounts together. (There are different ways this could be done – discuss alternative methods if necessary.)

R1,20						
R1,20						
R1,20						
R1,20						

Total: R33, 60 (28 x R1, 20 = R33, 60)

Questions on lesson 24

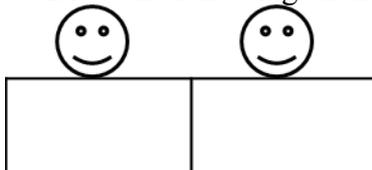
- 2.1 When would you use the strategy of tables and grids to solve problems?
- 2.2. Explain how the strategy of tables and grids leads to learners' understanding of multiplication?
- 2.3. With regard to the grid, explain alternate ways to calculate all the R1, 20 amounts together.

3. Problems solving related to fractions.

Participants work in pairs to solve the problem. Participants then answer the questions related to fractions.

Lesson 28: Sharing leading to fractions

- Share 12 counters equally between two learners
 - Give each learner a sheet of scrap paper and 12 counters.
 - Draw the following on the paper.



- How many counters do you each have?
- What is one half of 12?

ii. Do the same with one third and one quarter.

- Learners can draw what they find on scrap paper and you should discuss these drawings with the whole class.
- Thirds: Share the 12 counters equally between three learners

How many counters did each child get?

What is one third of 12?

- Quarters : Share 12 counters equally among four learners

How many counters did each child get?

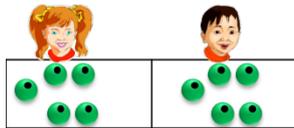
What is one quarter of 12?

iii. Ask the learners to show you the following

We are five friends. We have 20 counters. *If we divide them equally, how many counters will each of us get? What fraction will each friend get?*

Iv Look at the picture and answer the questions:

- How many marbles will each child get?
- What fraction of the marbles does each child get?



v. Twelve balls divided equally between four friends.

How many balls will each get?

What fraction will each friend get?

vi. Sixteen balls divided equally between two friends.

How many balls will each get?

What fraction will each friend get?

Lesson 30

i Tell the following “story” to the learners, while drawing the pictures on the board.

- Two friends share three cupcakes equally. 😊 😊 🍰 🍰 🍰
- *How many cupcakes will each friend get?*

ii. Draw the following pictures on the board. Learners need to make up the story and give the answer.



iii. Also do the following: six learners and nine cupcakes.

Questions related to problem solving fractions

3.1. Why do you think asking the right questions is important when teaching fractions?

3.2 How do fractions lead to division?

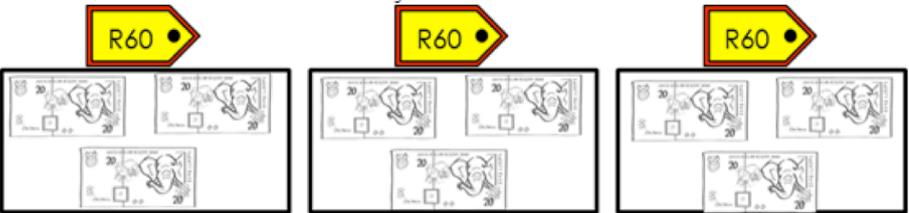
GRADE 3: There are five questions with sub questions to be done in 25 minutes

Participants work with a partner to solve the problems and then answer the related questions. Responses will be discussed at appropriate times during the activity.

1. Lesson 28: Solve the problem in question (a) below.

a) Linda bought 3 books for R60 each. How much change will she get if she has R200?

- *What is the key word?*
- *What is the question?*
- *What are the numbers?*
- Make use of banknotes to show your answers.



Questions on lesson 28

1.1 What do you think is the purpose of the three questions included in the lesson 28?

1.2 What prior knowledge do you think learner will need to solve the problem?

2. Lesson 7: Using technique of breaking down

Participants solve the following problems using the technique of breaking down

$128 + 214 =$	$438 - 323 =$
<p>i Jabulile read 425 pages. Buhle read 46 pages. How many pages did Buhle and Jabulile read altogether?</p> <p>ii Mrs Zuma needs to buy tiles for her bathroom. She needs 178 black tiles and 283 white tiles. How many tiles does she need altogether?</p>	

Question on Lesson 7

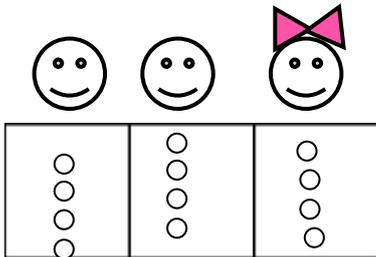
Explain how the concept of “regrouping” differs from “exchange”

3. Sharing leading to fractions

Lesson 31: Share 12 counters among 3 learners

Practice these exercise with your partner. Discuss with your partner the type of questions asked. How do the questions differ from each other? Why are these questions important? Why is it important to give learner opportunities to practice many examples?

- i Give learners 12 counters. Tell them to draw faces of three learners (2 boys and 1 girl) and to share the counters one at a time equally amongst the three learners. They use their Scrap paper/white boards to write on, e.g.



- *How many counters will each child get?*
 - *What fraction will the girl get?*
 - *How many will the girl get?*
 - *What fraction did the boys get?*
 - *How many will the boys get?*
 - *What is one third of 12?*
 - *What is two thirds of 12?*
- ii Repeat the above steps, asking the same questions, with the following examples:
- Share 12 counters equally among three boys and one girl
 - Share 12 counters equally among one boy and one girl
- iii Draw pictures to calculate.
We are five friends; two boys and three girls. We share 20 counters equally. How many counters will each friend get?
- *What fraction will each friend get?*
 - *What is one fifth of 20?*
 - *What fraction will the boys get?*
 - *How many counters will the boys get?*
 - *What fraction will the girls get?*
 - *How many counters will the girls get?*
 - *What is three fifths of 20?*
 - *What is four fifths of 20?*
 - *What is five fifths of 20?*
- iv Sharing and grouping
- Give learners counters to help them to work these calculations out practically and cups/containers to hold each person’s share. Divide the 9 counters equally between two

boys and one girl. Ask:

- *How many parts will you divide the whole into?*
- *How many counters will each child get?*
- *What fraction will the girl get?*
- *How many counters will the girl get?*
- *What fraction will the boys get?*
- *How many counters will the boys get altogether?*

Question on fractions

Compare the type of questions asked in grade three to the questions asked in grade two on fractions. How does it show progression in the levels of complexity?

4. Problem solving related to multiplication and division

Participants complete the exercise on multiplication and answer the questions

Lesson 14

The use of arrays to solve problems

- Draw an array on the board with 2 columns and 10 rows.
- How many circles are there in each row?
- How many circles are there altogether? Count:
- Let us write this as an addition number sentence.
- Let us write this as a multiplication number sentence.
- The inverse of multiplication is division.
- What would a division number sentence using this array look like?

Here are additional examples of using arrays to solve problems

- Draw an array to calculate. I want to make 4 cakes and for every cake I need 3 cups of flour.
- How many cups of flour do I need?
- Draw this as a rectangular array
- Write it as an addition number sentence:
- Write it as a multiplication number sentence:

Question on lesson 14

What is your understanding of arrays and what is the purpose of arrays in mathematics?

5. Word problems involving repeated sets.

Lesson 32

A vegetable garden has 4 rows of plants. Each row has 2 plants. How many plants are there in the garden?

- Let us write it as an addition number sentence:
- We can say there are 4 rows with 2 plants in each row. Draw a picture if necessary.
- Let us write it as a multiplication number sentence:
- Ask learners to make up other stories that lead to multiplication – where repeated sets are involved. E.g. A car can take 5 passengers. How many passengers can 2 cars take? ($5 \times 2 = 10$)

My dad planted 5 fruit trees in a row. He planted 6 rows. Follow the same steps as above

- How many fruit trees did he plant?
- Write it as an addition number sentence:
- Write it as a multiplication number sentence:
- Make up other stories that lead to multiplication where repeated sets are involved.

If I have 42 biscuits and I share them between 3 learners, how many biscuits will each learner get?

- Write this as a number sentence:
- Tell another story about the division number sentence.
- Do the same for 27, 39, 48, and 54. If you share each of these numbers of biscuits between three learners, how many biscuits will they get (each time)?
-

Question on lesson 32

Identify the strategies that lead to an understanding of multiplication and the strategies that leads to an understanding of division

H. REFLECTIVE PRACTICE IN THE CONTEXT OF LEARNERS' PROBLEM SOLVING

[You will spend 30 minutes on Section H]

Input and discussion

Reflection refers to thinking about what you have done in the classroom, thinking about why you did it, and thinking about if it worked – it is a process of self-observation and self-evaluation. By collecting information about what goes on in our classroom, and by analysing and evaluating this information, we identify and explore our own practices and underlying beliefs. This may then lead to changes and improvements in our teaching especially if we share our reflections with our peers, and together try to solve problems we are facing.

One of the most valuable sources of evidence of 'how well we are doing' is learners' work. Learners' oral responses and questions in class, and their written work tell us a great deal of what they have learnt. Analysing learners' work, thinking about the reasons for any errors and misconceptions they demonstrate, and then thinking about how we taught the work and how we could have done it better is one way in which we can improve our practice.

Activity 9: Analysing and reflecting on learners' work

The tables below (on the next page) show responses of of three grade three learners which were scanned from the ANA 2013 diagnostic report. Study the responses carefully and consider the questions which follow.

10.1	Count forwards in 100s	584	585	586	587	588	A
10.2	Count backwards in 20s	320	310	320	330	240	

Complete the table:

10.1	Count forwards in 100s	584	585	586	587	588	A
10.2	Count backwards in 20s	320	321	322	323	240	

Complete the table:

10.1	Count forwards in 100s	584	4	5	8	100	A
10.2	Count backwards in 20s	320	3	2	0	240	

1. What skills and knowledge would learners need to be able to answer this question correctly?
2. Read the information below or listen to the facilitator's input about errors and slips. Then answer the question that follows:

When we talk about work that learners got wrong we use words such as mistake, error or slip. Errors tend to be systematic, persistent and pervasive mistakes performed by learners across a range of contexts. They are based on conceptual misunderstanding. Slips are mistakes that are easily corrected when pointed out. Slips or mistakes are more like careless errors. Since errors are systematic and persistent, they are not necessarily responsive to easy correction or re-explanation of concepts. When doing error analysis, teachers need to examine all learner work – both errors and slips, so that they get an overview of what their learners know/don't know and how to address content issues that is evident in their learners' work. They also will realise how deeply embedded the problem is (error) or not (slip).

Question: Identify the mistakes made by each learner and state whether it's an error or slip

3. What are some of the reasons attributed to learners' errors, slips and misconceptions?
4. Why is it important to think about why errors you observe in learners' work have been made?
5. What does this activity (your error analysis) teach you about the way you would reteach the content in the item?