FACILITATOR'S GUIDE

2018 TRAINING WORKSHOP NO.2

MATHEMATICS

FOUNDATION PHASE

PILO

IMPROVING LEARNER PERFORMANCE

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Foundation phase
Just-in-Time Training Workshop
2018: No.2

Facilitator's Guide

MATHEMATICS

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what I do matters

Endorsed by:

SADTU NAPTOSA NATU

Jika iMfundo
Foundation Phase JIT 2 of 2018
Mathematics
Workshop guide for facilitators

In this workshop participants will find out more about:

- Problem solving using the Jika iMfundo Toolkit
- How to teach problem solving in mathematics

Participants will work in groups on all of the activity questions. Time guidelines are given and the facilitator will interact with the participants while they work. Participants will also be able to share key ideas together with the large group

Materials:
Participants will need the following materials

1. Term 3 lesson plans and trackers for Grades 1 to 3. Extracts that are relevant to each grade are included in the participants' hand-out.
2. Counters (about 50 counters per group of 5 participants)
3. Paper for participants to work on

SUGGESTED WORKSHOP PLAN		
Time	Duration	Session/Topic
8H00 - 8H30	30 minutes	o Arrival/ Register o Distribution of workshop material o Introduction to the workshop
8H30 - 13H30	5 hours	**Problem solving** o The need to teach problem solving o What are problems? Types of problems and broad approaches to teaching problem solving o Problem solving in the CAPS for FP o Types of knowledge required to solve problems in Maths o Mathematical proficiency o Developing basic operations through problem solving o Problem solving using different operation strategies in the Jika iMfundo lesson plans o Reflective practice in the context of learners' problem solving **Note:** A tea break will be taken at a convenient point in the workshop.
13:30 – 13: 40	Closure	
13: 40	Lunch	
Total working time	4.5 hours	

A. The need to teach problem solving (You will spend 20 minutes on section A)

Performance by South African learners in the national systemic evaluation (DoE) as well as in regional SAQMEQ and international assessments suggests that learners are not thinking mathematically. An analysis of the SACMEQ and TIMSS test items reveals that teachers emphasise the teaching of calculation, algorithms and procedures rather than helping learners "think mathematically". While it is critical for learners to perform basic operations, to know their basic number facts, and to perform mental arithmetic with confidence, these alone are not enough. Learners need to understand the mathematics they learn in flexible and meaningful ways so that they can apply it with confidence and make sense of the world. This can be contrasted with the traditional, stereotypical way of teaching in which the teacher explains the rules, provides an example and then drills the learners with similar examples

Activity 1: Think about your own problem solving (10 minutes)

Getting yourself to a training workshop in the course of a busy week presents problems that you need to solve. What problems arose for you and how did you resolve them in order to be here at today's workshop?

Response to Activity 1

Participants will note different ideas – circulate and listen to their discussion, noting key points that will reinforce the discussion on maths problem solving that follows this activity.

- *Identify key issues to solve.*
- *Plan strategy for solution*
- *Get going and be sure to arrive on time*

Point out that many real life problems have more than one solution depending on varying circumstances. As in a real life model, mathematical problems too can have multiple solutions. We need to include ambiguity in problems: not all problems should be simple and straightforward, as this does not equip our learners for the ambiguities that arise in real life.

Problem solving trains us for the real life. Our mathematics training can be seen as equipping us for everyday situations and some of the problems we are confronted with in life. We need to approach a problem systematically. Consider the following steps which could guide you towards successful problem-solving.

Step 1: Read the problem carefully and ensure that you understand what the problem is about. Restating the problem in your own words is a good exercise, which will make it clear to you whether or not you have understood the meaning of the problem. It is often a good idea to try and sketch a diagram that assists you to illustrate what is required by the problem.

Step 2: Once you have understood what the problem is asking, you have to think of your strategy for solving the problem. Think about whether you have all the information that you need in order to solve the problem? Have you solved other similar problems which can guide your solution to the current problem? And, can the problem be broken up into smaller parts if it seems too big to solve all at once?

Step 3: Here you go about implementing your problem-solving strategy to get to the actual solution to the problem. It is important that you realise the difference between devising a strategy to solve a problem and the actual solution to the problem. Both are important activities. It will become clear to you if you need to change your strategy or find a new one, or if you're original strategy was adequate.

Step 4: Once you have solved the problem, a final "logic check" of your solution is never a waste of time. Careless errors can slip into your working (though your strategy may be correct) and lead you to an answer which is not correct. Re-read your work just to be sure that it makes sense and presents a valid, satisfactory solution to the problem. This step of verification may seem like a waste of time, but will often prove its usefulness when on verification; you make small changes and improvements to your answers.

Activity 2: Summarise the key steps in solving a problem (10 min)

How would you summarise each of steps 1 to 4 above in one sentence or phrase?

Step 1: *Reading the problem*
Step 2: *Comprehending the problem*
Step 3: *Implementing the strategy*
Step 4: *Solving the problem and checking solution*

B. What are problems? Types of problems and broad approaches to teaching problem solving

[You will spend about 30 mins on Section B]

What is a problem?

Hiebert, et al. (1997, 75) explain a problem as a task or activity for which learners have no prescribed or memorized rules or methods. There is no specific 'correct' solution method. To find a solution, learners must draw on their knowledge, and through this process, they will often develop new mathematical understandings

The following are important points to note about problem solving:

- A task does not have to be a word problem to qualify as a problem — it could be an equation or calculation that students have not previously learned to solve.
- Also, the same task can be a problem or not, depending on when it is given. Early in the year, before learners learn a particular skill, the task could be a problem; later, it becomes an exercise, because now they know how to solve it.

There is a distinction between *teaching problem solving* and *teaching through problem solving*

Figure 1 below summarises the key features ***of teaching problem solving*** *and* ***teaching through problem solving***

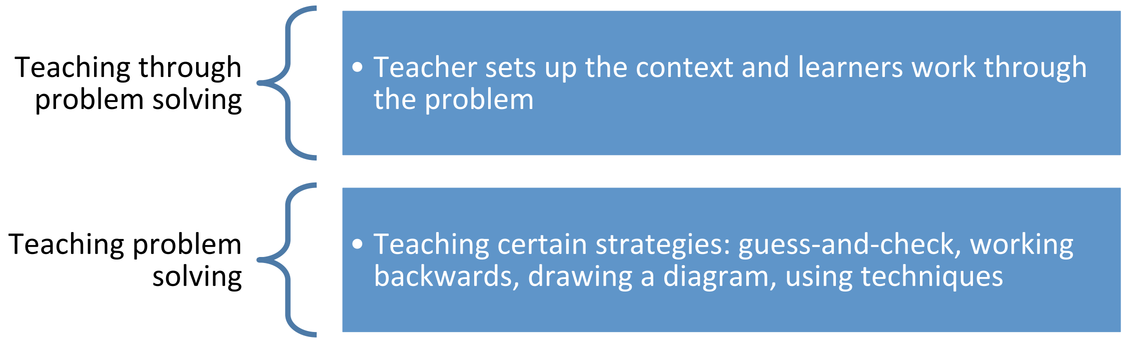


Figure 1: Teaching *through problem solving* and *teaching problem solving*

Teaching problem solving: Focuses on teaching certain strategies — guess-and-check, working backwards, drawing a diagram, and others. In a lesson about problem solving, learners might work on a problem and then share with the class how using one of these strategies helped them solve the problem. Other learners applaud, the learners sit down, and the lesson ends. These lessons are usually outside the main flow of the curriculum; indeed, they are purposely independent of any curriculum

Teaching through problem solving: A "teaching through problem solving" lesson would begin with the teacher setting up the context and introducing the problem. Learners then work on the problem for about 10 minutes while the teacher monitors their progress and notes which learners are using which approaches. Then the teacher begins a whole-class discussion. Similar to a "teaching problem solving" lesson, the teacher may call on learners to share their ideas, but, instead of ending the lesson there, the teacher will ask learners to think about and compare the different ideas — which ideas are incorrect and why, which ideas are correct, which ones are similar to each other, which ones are more efficient or more elegant

Types of problems
Problems are categorised as routine and non- routine problems as seen in Table 1 below

Table 1: Difference between routine and non-routine problems

Routine Problems	**Non Routine problems**
• Use known or prescribed procedures in obtaining solutions • Use traditional worded problems to enable students to use standard algorithms • The problem to be solved is similar to one that has been done before. Algorithms can be seen as: rules for calculating computational procedures logical step by step procedures	• Use more than one strategy or solution • No algorithms exist • Solved through the use of heuristics and not algorithms **Heuristics are:** • Procedures or strategies that do not guarantee a solution • But provides a possible way to discover a solution E.g. Building a model or drawing

Secondly, problem types are categorized as **"problems in context" and "context - free calculations (CAPS).** It is important for teachers to know the distinction between "**Problems in context**" and "**Context free calculations**" as described in CAPS as this will help teachers to ensure that they include all types of problems. Figure 2 below represents the distinction.

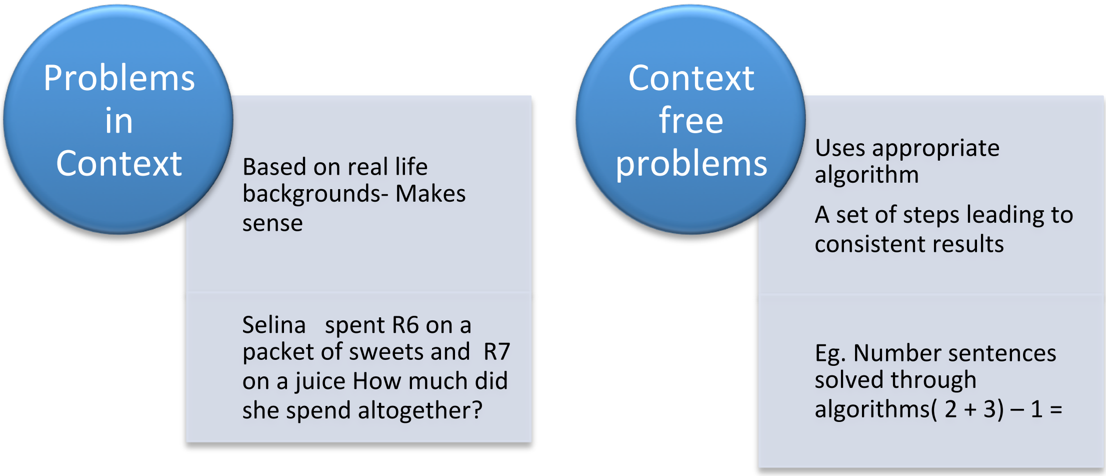


Figure 2: Problems in context and context free problems

Activity 3: Compare routine and non-routine problems (10 minutes)

Look at the examples of a non-routine and a routine problem and then discuss with the person sitting next to you the features of non-routine problems. You may compare this to the routine problem. Is it a straightforward question? Is there just one answer? Is there any one specific method to get to the answer?

Non routine problem

If hot dog rolls come in bags of 8 and viennas are sold in packs of 12, what is the least number of each pack that a person must buy in order to have 1 vienna for every roll? List other numbers that will ensure that a person has 1 vienna for every roll without any remainders.

Routine problem

Mum bought a pack of 10 viennas and a dozen rolls. How many more rolls are there than viennas?

Response to activity 3

Answer: 3 packs of viennas and 2 packs of rolls. So we know that for every 3 packs of viennas you will need 2 packs of rolls. Therefore there could be many correct answers.

5 and 4; 8 and 6

In the non-routine problem:

- *No clear or prescribed method or approach to arriving at the solution is implied in the question.*
- *The problem is novel and no approach is specified or readily identifiable; hence, the problem solver will have to devise his or her own method of obtaining an answer.*
- *The question engages learners in critical thinking. They think about and attempt to develop a strategy to obtain a solution; they draw conclusions, inferences, conjectures, and develop and test hypotheses.*
- *Learners' solutions may vary. While all the answers are expected to be the same,*
- *Learners' strategies may differ. Some solutions will be more sophisticated than others –*
- *Some learner may try many approaches and not arrive at an answer, while others may*
- *Find methods that work to obtain an answer and to gain insight into the problem.*
- *The task encourages communication and collaboration. Given that no clear procedure*

exists for solving the problem, it is likely that learners will have to collaborate with each other and engage themselves in mathematical discussion as they seek solutions and attempt to communicate these results for others to understand.

C. Problem solving in the Foundation Phase according to CAPS

[You will spend 40 minutes on section C]

In this section, teachers will develop their understanding of problems in context relevant to foundation phase teaching as stated in CAPS. Teachers will refer to Table 2 below to answer questions to demonstrate their understanding of the policy requirements related to "Problems **in context**"

Table 2: Solving Problems in context (Adapted from the Foundation phase overview (CAPS, 2011: 20)

For easy referencing, the numbering in this table is according to CAPS.

Solving problems in context	**Grade 1**	**Grade 2**	**Grade 3**
1.6. Problem solving techniques	Use the following techniques when solving problems and explain solutions to problems: • Use concrete apparatus e.g. counters • Draw pictures to draw the story sum • building up and breaking down number • doubling and halving • number lines supported by concrete apparatus	Use the following techniques when solving problems and explain solutions to problems: • drawings or concrete apparatus e.g. counters • building up and breaking down number • doubling and halving • number lines	Use the following techniques when solving problems and explain solutions to problems: • building up and breaking down number • doubling and halving • number lines • rounding off in tens
1.7 Addition and subtraction	Solve word problems (story sums) in context and explains own solutions to problems **involving** addition and subtraction with answers up to 20	Solve word problems (story sums) in context and explains own solutions to problems **involving** addition and subtraction with answers up to 99	Solve word problems (story sums) in context and explains own solutions to problems **involving** addition and subtraction with answers up to 999
1.8 Repeated addition leading to multiplication	Solve word problems (story sums) in context and explains own solutions to problems **involving** repeated addition with answers up to 20	Solve word problems (story sums)in context and explains own solutions to problems **involving** repeated addition with answers up to 50	Solve word problems (story sums)in context and explains own solutions to problems **involving** repeated addition with answers up to 100
1.9 Grouping and sharing leading to division	Solve and explain solutions to practical problems **involving** equal sharing and grouping with whole numbers up to 20 and with	Solve and explain solutions to practical problems **involving** equal sharing and grouping with whole	Solve and explain solutions to practical problems **involving** equal sharing and grouping with whole numbers up to 100 and with

	answers that may include remainders	numbers up to 50 and with answers that may include remainders	answers that may include remainders
1.10 Sharing leading to fractions		Solve and explain solutions to practical problems **involving** equal sharing leading to solutions that include unitary fractions	Solve and explain solutions to practical problems **involving** equal sharing leading to solutions that include unitary and non-unitary fractions

Activity 4: Interpreting the extract from CAPS. (20 mins)

This activity has 12 questions, based on Table 2

1. What is meant by "problem solving techniques" as stated in 1.6 in Table 2? Give some examples of problem solving techniques.

Problem solving techniques refer to the strategies used in solving the problems e.g. drawing, using counters, breaking down numbers, halving/ doubling. As number range increases up to grade 3, learners should develop more efficient strategies for calculations. The teacher needs to take into account the number range as well as the calculation competencies for context based problems

2. What knowledge and skills will learners need to "explain solutions to problems"?

Correct mathematical terminology, language, number operations, ability to calculate.

3. Compare the problem solving techniques used in Grade 1, 2 and 3

- *Grade 1 and 2 they draw and use concrete objects, Grade 3 no drawing and concrete objects*
- *Building up and breaking down numbers are done in all three grades*
- *Doubling and halving in all grades*
- *Grade 1 number line is supported with concrete objects. Grade 2 and 3 – only numerals*
- *Rounding off is only in grade 3*

4. Explain how you understand the outcome: "Explain solutions in context **involving** addition and subtraction". What does the word "involving**"** mean?

What should be noted is that the curriculum talks about "solving problems that involve…" as opposed to listing addition, subtraction, multiplication and division as ends in themselves. It should also be noted that the curriculum mentions problems involving sharing and grouping before it mentions problems involving addition and subtraction – this is not co- incidental. Problems involving sharing and grouping are mentioned before problems involving addition and subtraction expressly because young learners relate more easily to situations that involve sharing (and grouping) than they do to situations that involve addition and subtraction.

5. What does it mean to "find a fraction of a whole"?

This is the concrete way of finding fractions – when the fraction takes on a visual form rather than simply being an abstract number. Learners start to learn about fractions by finding fractions of wholes. This teaches the idea that a fraction is a "part of a whole", that is not a whole number.

Every time they find a fraction part it must be emphasized that the parts of the whole must be ***equal in size*** *in order for them to be fraction parts.*

6. Explain how the problems related to “Sharing leading to fractions” differ in each grade.
   - *Grade 1- Sharing leading to fractions problems are not done*
   - *Grade 2- Unitary fractions*
   - *Grade 3- Non unitary and unitary fractions*

7. The curriculum uses the terminology “unitary” and “non-unitary”. Give some numeric examples of “unitary” and “non-unitary” fractions.

A fraction which has a numerator value of 1 is a unitary fraction. For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, etc.$
A non-unit fraction is a fraction where the numerator (the number on the top half of the fraction) is greater than 1. For example, 3/4 is a non-unitary fraction, because three, the numerator, is greater than 1.

8. How does sharing leading to fractions differ across the grades?

The number range increases.
In all grades, sharing and grouping includes remainders

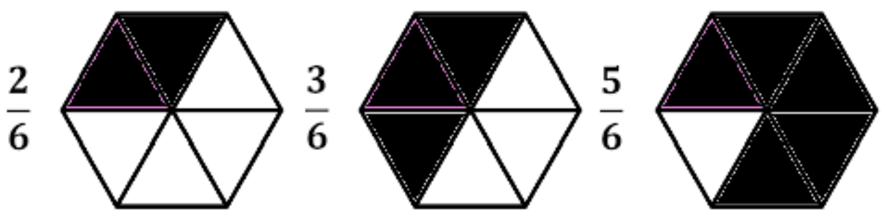
9. What is meant by the phrase “problems that involve equal sharing”?

Equal sharing means sharing into parts that are equal in size or number.

10. Why is equal sharing essential in the context of fraction concept development?
Initial examples of sharing that lead to finding fractions involve equal sharing because this stresses the idea of the equality of the parts that make up the whole, when we find fractions.

11. 11. Which of the following are examples of “unitary” fractions and which are examples of “non-unitary” fractions? What makes them different?

These are unitary fractions, in each case only one part is shaded and so the fraction has a numerator of 1.



These are non-unitary fractions, in each case more than one part has been shaded and so the fraction has a numerator of more than 1.

12. Why do you think it is important for teachers to have a good understanding of Table 2?

To plan appropriate activities and problem solving types. To understand progression and expected prior knowledge for interventions.

The next sections will focus on the different types of knowledge and the five strands of mathematical proficiency. The two aspects are related to mathematical thinking which is the cornerstone of problem solving as stated in CAPS

Problem solving should be the goal of all mathematics instruction and an integral part of all mathematical activity. Learners should use problem solving approaches to investigate and understand mathematical content and to help learners think mathematically.

BUT

The challenge for all Numeracy teachers is to answer the question:

How do I teach in a way that will help children to think mathematically?

D. Types of knowledge required to develop problem solving (20 minutes)

To help learners to think mathematically, it is imperative for the teacher to have a thorough understanding of the different kinds of knowledge, namely physical, social and conceptual (See Figure 3) and the five strands of Mathematical proficiency, namely understanding, applying, reasoning, engaging and computing which will be returned to later.

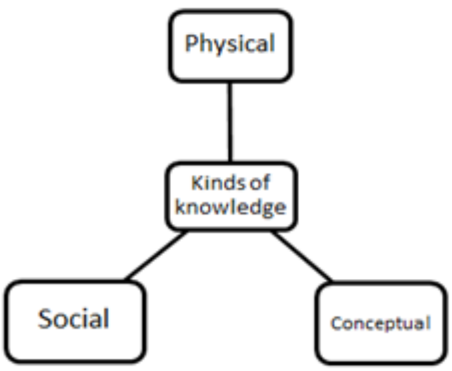


Figure 3: Types of knowledge

- Physical knowledge is derived through touching, using and playing with concrete materials.
- Social knowledge refers to knowledge that needs to be told to people and remembered by them as social knowledge
- Conceptual knowledge is knowledge that is constructed internally by each individual by themselves

Piaget distinguished between the different types of knowledge- Physical, social and conceptual. When thinking about how learners earn mathematics, it is important to consider these different knowledge types and how they are related. All three are inter-related. Learning experiences for young learners should emphasize constructive knowledge, and not transmission. Construct learners' understanding and knowledge, by helping them to become thinkers. Table 3 gives some of the key characteristic of each kind of knowledge.

Table 3: Characteristics of three types of knowledge

Physical knowledge	Social knowledge	Conceptual knowledge
• Acquired through interaction with the physical world e.g. through touching, observing and handling objects • Learners need concrete experiences to develop physical knowledge • Problem solving- When learners draw, they use physical understanding of the problem • Implications- Mathematics classroom must contain lots of concrete apparatus	• Knowledge that needs to be told by people and remembered • Conventions and rules are examples • The words add, subtract, minus and the symbols used to represent them • Teacher has to distinguish between knowledge that must be told and knowledge that must be constructed. Knowledge that can be developed through constructivism should not be told	• Internal knowledge, constructed by each individual themselves • Teacher creates activities and situations, encourages learner to reflect on what they are doing • Children must express their thoughts in words, actions and methods • Cannot be taught through direct instruction

Activity 5: **Planning activities that develop different kinds of mathematical knowledge (15 minutes)**
This activity has 4 questions to complete. You should spend about 5 minutes on each question

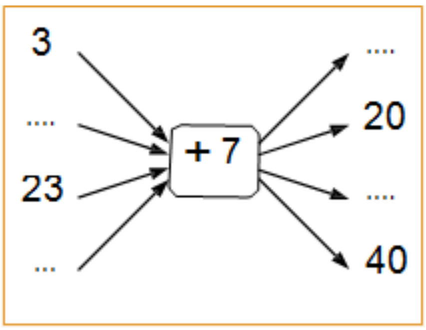
1. Participants are divided into 5 groups. Each group is allocated a topic (Numbers operations and relationship, patterns, space and shape, measurement and data handling). The groups plan different activities that include different type of knowledge, i.e. physical, social and conceptual, to teach any concept related to the topic that is assigned to them

Response to question 1: The following are examples of activities that could be considered

Types of knowledge	*Examples of activity*
Physical	1. *Number: Counters, body parts, concrete materials* 2. *Patterns: Copying and extending patterns using matchsticks, tiles, blocks and other apparatus that learners develop a sense of behaviour of patterns and learn to make predictions* 3. *Shapes: Building objects; Covering (tiling) surfaces; and making new shapes and objects that learners develop a sense of the relationships and properties of the shapes and objects that they are working with* 4. *Measurement: Informal measurements using strings, cups, object, etc. Through measuring they also meet up with situations which cause them to think about the need for parts of a whole to describe certain quantities – an important introduction to the concept of fractions.*

	5. *Data: Collecting and sorting of physical objects that learners develop their first sense of what it means to work with data.*
Social	*knowledge that needs to be told to people and remembered by them as social knowledge* *For example teachers need to teach learners:* • *Vocabulary such as: Number names;* • *The names of shapes and objects; and* • *The words that we use to describe operations. Learners can perform the "basic operations" without any knowledge of the words "addition, subtraction, division and multiplication". It is the role of the teacher to help learners develop the language with which to describe their activities.* *Includes conventions such as:* • *The way in which we write the number symbols;* • *The way in which we write a number sentence to describe problems; and* • *The way in which we use the equal sign to denote equivalence.*
Conceptual/ Logico	*Needs to actively encourage learners to reflect on what they are doing and what they are thinking. The teacher must help learners to verbalize their observations so that they can explain these to the other learners as well as learn to interpret the explanations of the other learners.* *When designing a lesson/task; the teacher needs to ask the question:" What do I want learners to learn from this situation? The teacher then needs to shape the situation/problem/activity in a way that will invoke learners to "see" the patterns and structures. Furthermore, both during and on completion of the activity, the teacher needs to facilitate reflection on the activity by the child. It is this reflection more than anything else that will support the development of conceptual knowledge*

2. "Completing tens (or hundreds)" is an important skill that learners need to develop in order to break down and build up numbers with confidence. To support learners in developing this skill a teacher might ask her class to complete a number of flow diagrams such as the one below. The teacher wants the learners to observe the pattern e.g. "Adding a seven to a number ending in 3 completes the ten". **Please complete questions a to d in the box below**.



Complete the calculation in the spider diagram. Then answer the questions that follow:

a) After completing the answers, describe the pattern you identified.

b) Can you apply this pattern to another example?

c) How do you think this activity can lead to conceptual development?

d) "Telling learners the rule" makes it easier for learners to learn than asking learners to identify patterns and apply patterns. To what extent do you agree with this statement?

1

Response to question 2.

a) When you add 7 to a number ending with 3, the answer is a whole ten. If you subtract 7 from a whole ten number, the answer ends with a 3.

b) Allow teachers to share their own examples e.g. Adding an 8 to a number ending in 2 completes tens

c) The teacher will progress from physical knowledge, social knowledge then conceptual knowledge

Using concrete objects to learn what makes 10s. They then exchange the 10 one for a base ten which they then add to the tens.

1 and 9	*If I add 9 to any number ending with 1 I get 10*
2 and 8	*If I add 8 to any number ending with 2 I get 10*
3 and 7	*If I add 7 to any number ending with 3 I get 10*
4 and 6	*If I add 6 to any number ending with 4 I get 10*
5 and 5	*If I add 5 to any number ending with 5 I get 10*
6 and 4	*If I add 4 to any number ending with 6 I get 10*
7 and 3	*If I add 3 to any number ending with 7 I get 10*
8 and 2	*If I add 2 to any number ending with 8 I get 10*
9 and 1	*If I add 1 to any number ending with 9 I get 10*

d) Disagree. Conceptual knowledge means that learner construct this knowledge themselves. They must discover this for themselves

3. Study the two scenarios on counting below. Then identify the kinds of knowledge evident in each classroom. Provide a reason for your answer. Also consider the merits/ weaknesses of these strategies.

A. Ms Ncube taught her grade one learners the number names. She then asked the learners as a class to count orally (chanting number names) from 1 to 100. Thabo struggled at 29, and then he would say twenty ten, twenty eleven, etc. But as he listened to other learners count, he was then able to count properly.

B. Ms Dlamini asked her grade one learners to count the number of objects in a pile by touching each object as they counted. She then asked the learners to make groups of tens, some counted in fives.

Response to question 3

- *Social knowledge- Learners count together with other learners. They recite after hearing from others*
- *Physical knowledge – They derive knowledge through the use of physical objects by touching the objects.*
- *Conceptual knowledge when they could apply their grouping in twos as an efficient counting strategy.*
- *Social knowledge is less threatening. Facilitates co-operative learning*

4. Study the scenario on patterns below and answer the questions that follow:

> The learners in Ms Higg's class are determining the values of 18 - 9; 15 - 9; and 13 - 9. They were given several examples to practice with their concrete resources. Ms Higgs used two strategies:
>
> **Method 1:** Used numberline to teach technique of "rounding off". E.g. 18- 9 =... Round off 9 by adding 1 to get 10. So if they added 1 to 9 they add 1 to 18 to get 19 (Because what we do to the left, we must do on the right) 19 – 10 = 9.
>
> **Method 2**: Taught them a rule: A quicker way is to remember the easy rule: "When we subtract 9 from 18 we add the 1 and the 8 to get 9 – this is our answer. Similarly when we subtract 9 from 15 we add the 1 and the 5 to get 6 – this is our answer.

i) Identify the type of knowledge in above methods. Provide a reason for your answer.

ii) Which method do you think it most effective and why? What could be a limitation of this method?

Response to question 4

> *i. Method 1 shows physical knowledge- used numberline and conceptual knowledge. The learners have reached the level of conceptual understanding only after practicing these examples several times. They identify their own rules. Method 2 shows Social knowledge, Learners were told the rule. Also procedural knowledge.*
>
> *ii. First method is most effective. Learners can derive meaning. However when learners are given the rules without first exploring and solving for themselves, the rules could be problematic. Firstly learners have to remember that these rules only apply to subtracting 9. They will not understand why this rule only applies when the ten value is a 1 ten.*

Next, let's look at what it means to be mathematical proficient.

E. Mathematical Proficiency [Spend 20 minutes on section E]

What does it mean to do mathematics?

A teacher's understanding of what it means to do mathematics influences how she will teach mathematics. If a teacher's understanding of doing mathematics involves memorising facts, rules, and procedures to determine the answers to questions then she will teach in a particular way. By contrast, if a teacher regards doing mathematics as a sense-making problem-solving activity then her teaching approach will be quite different. Doing mathematics is more than getting the answers right. It is about being mathematically proficient.

Mathematical proficiency is key in developing a strong mathematical learning. Kilpatrick, Swafford and Findell (2001) define mathematical proficiency as having five intertwining strands as can be seen in Figure 4 below.

The development of mathematical proficiency takes time. In each grade, learners need to make progress along every strand. Each strand is important and interwoven with the others. Learners are described as mathematically proficient if they can demonstrate all five of the competencies as seen in the Figure 4 below:

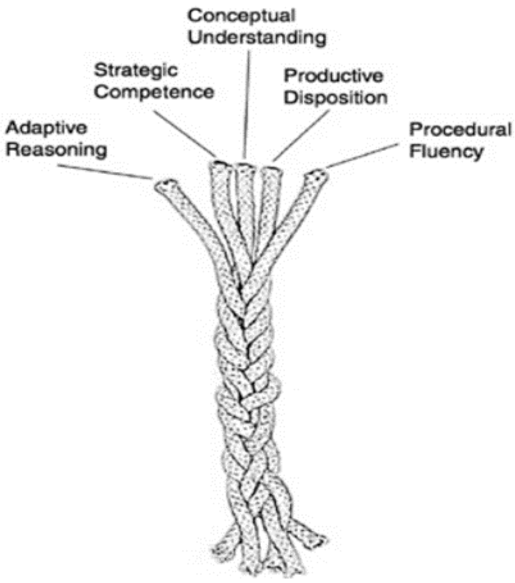


Figure 4: Mathematics Proficiency

• Conceptual understanding—an understanding of concepts, operations and relations. Learners are able to comprehend connections and see similarities between interrelated facts.

• Procedural fluency—flexibility, accuracy and efficiency in implementing appropriate procedures. This includes efficiency and accuracy in basic computations.

• Strategic competence—the ability to formulate, represent and solve mathematical problems. This is similar to problem solving. Strategic competence is mutually supportive with conceptual understanding and procedural fluency.

• Adaptive reasoning—the capacity to think logically about concepts and conceptual relationships. Reasoning is needed to navigate through the various procedures, facts and concepts to arrive at solutions.

• Productive disposition—positive perceptions about mathematics. This develops as students gain more mathematical understanding and become capable of learning and doing mathematics.

An understanding of these strands will enable teachers to teach for mathematical proficiency.

Activity 6. Mathematical proficiency. This activity has 2 questions. (10 minutes)

1. Why it is important for teachers to have a good knowledge of the five strands of mathematical proficiencies.

Response to question 1

They remind teachers that teaching mathematics is much more than the teaching of methods or procedures only. Teaching mathematics involves helping learners to understand, to apply, to reason and to adapt

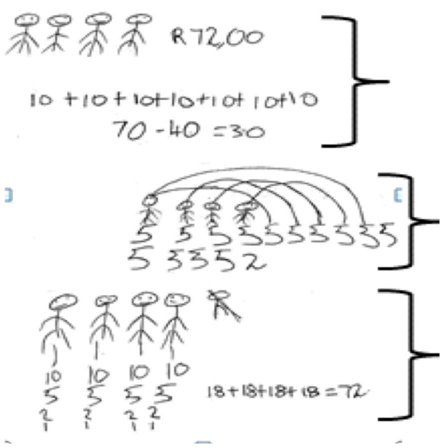
They allow teachers to reflect on their teaching "Am I doing the right thing?" If the learners in your class are willing and able to apply their mathematical knowledge with understanding to solve non-routine problems and justify their solution method(s) they are becoming numerate/mathematically proficient.

2. Answer the questions in the box beside Lerato's response below.

Lerato (Grade 2) is solving the problem: 4 learners are paid R72 altogether. If they share the money equally between themselves, how much will each person get?

Lerato's Problem solving:

Grade 2



1. What strand of mathematical competency is reflected in Lerato's response? Provide a reason
2. Analyse the different steps used in the process and discuss what you think Lerato was thinking during the process.

Response

1. *Reasoning, because she tried different strategies. Lerato drew 4 children and started by giving each child R10. Then she realized she had more but couldn't give another 10 to each child. So she gave 5. Then she gave 2 each, then 1 each. She then added 10, 5, 2 and 1 to get 118. She added 18 four times and got 72*

F. Developing basic operations through problem solving [Spend 40 minutes on section F]

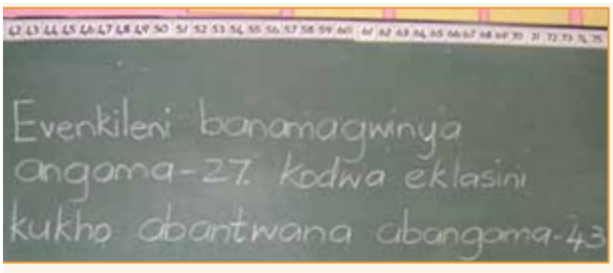
In this section, participants will find out more about how basic operations such as addition, subtraction can be developed through problem solving

Activity 7. Analyse problems and learner responses involving number operations (20 minutes for activity)

Answer 4 questions in this activity

1. Study the responses of two learners to the problem about amagwinyas. Discuss in your group how the problem was used to teach number operations. Also compare the two approaches used by the learners.

The grade 2 learners were given the problem: The tuck-shop has made 27 amagwinya (vetkoek). There are 43 learners in the class. Are there enough amagwinya for each child to get one? After some class discussion it was agreed that there were not enough amagwinyas and the teacher asked the learners to determine how many more were needed.



The following were the responses from two learners, namely Odwa and Andrew

Odwa solved the problem by first drawing 27 stripes to represent the 27 learners in the class and then he drew a large number of extra stripes. He counted on from 27: 28, 29, 30 … 43 and highlighted the 43rd stripe. Finally he counted how many extra amagwinya (stripes) were needed and concluded that 16 more amagwinya needed to be made. We can summarise Odwa's method as follows: **27 + 16 = 43**	
Andrew solved the problem by first counting out 43 counters. Next he counted out 27 from the 43 – as if he was giving amagwinya to those who he could give to. Finally he counted the remaining counters and established that he still needed 16 amagwinya for the remaining learners. We can summarise Andrew's method as follows**: 43 – 27 = 16.**	
The teacher then went on to explain the different approaches used by both Odwa and Andrew. Then only did she introduce the notations and what it meant.	

Response to activity on Amagwinyas

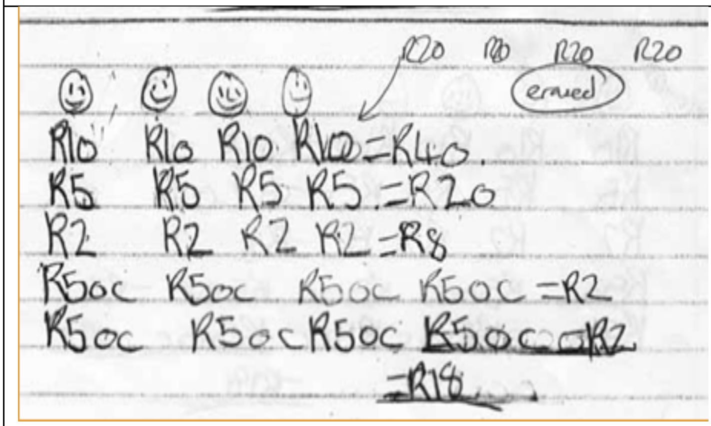
The amagwinya example illustrates a number of important ideas about the role of problems:

Learners can solve problems without knowing the words addition or subtraction or for that matter before they know the symbols representing these operations.

- *The amagwinya problem is not necessarily an addition or a subtraction problem but rather a problem.*
- *When learners are asked to solve problems by making sense of the situations as in the case of Odwa and Andrew they develop a sense of what it is to add, to subtract, to multiply and to divide from the problem rather than being told the meaning of these operations by their teacher – they are developing their conceptual knowledge.*
- *With time the teacher will teach the class that "Odwa's method" is more generally referred to as addition and uses a notation while "Andrews's method" is referred to as subtraction and uses a different notation.*
- *When the teacher introduces the names and the symbols for the operations she is introducing the learners to the social knowledge which is commonly used to describe the perfectly natural actions of the learners.*

2. The following problem was solved by two grade three learners (Mandy and Sally who used different approaches as illustrated below: Analyse their approaches and answer the questions that follow.

Brenda, Roleen, Vuyani and Casper help Mr Kumalo in his vegetable garden on a Saturday morning. The children all work just as long as each other. Mr Khumalo gives them R72 altogether. How must the children share this amount evenly among them?

	Responses to a. ***Feedback on Mandy's approach*** *In solving the problem Mandy drew the 72 Rands and the four learners. Careful analysis of Mandy's work shows that she starts out by giving each child R10 (notice how the first four rows of Rands are crossed out in lots of 10), realising that she does not have enough Rands left over to give each child another R10, she "halves" the number that she is giving to each child and so gives them R5 each, next she again "halves" and gives each child R2 and finally is left with R4 and gives each child one last Rand. She concludes that each child will get R18.*
	***Feedback on Sally's approach*** *By contrast Sally solved the problem by drawing four faces representing the four learners in the story. She started out by giving each child R20 – realising that she had given away more money than had in total she erased the R20 per person and tried instead R10 each. Next she "halved" and gave each child R5 and continues in this way until she had used up all R72 and also concluded that each child would get R18.*

a) Interpret what the learners were thinking as they solved the problem.
b) Why is it important for children to use illustrations when solving problems?
c) Why do you think it is important for teachers to study learner's illustrations in problem solving?

Response to question 2

a. See insertions in table above

b. Illustrations help learners to understand number operations and at the same time to develop a deeper sense of what it means to add, subtract, multiply and divide. . Learners can also refer to their illustrations when they discuss how they got the answers

c. It is important for the teacher to interpret learners' thinking in order to identify learning *gaps and provide appropriate support. Illustrations make learners' thinking explicit which helps teachers to see how learners are thinking. Teachers can adapt their instruction according to the way learners think.*

3. In this activity, you will notice that different approaches that may be used to solve a problem. Below are two sets of problems; set A and set B.

3.1 Work through the problems in SET A on your own.
3.2 Compare your approach with the approach used by the person sitting next to you. Try to identify the key differences in the various solutions. How might learners approach them?

3.3 Work through the problems in SET B on your own.
3.4 Compare your approach with the approach used by the person sitting next to you. You will realize that in all likelihood, the approaches used to get the solution differ. Try to identify the key differences in the various solutions. How would learners approach them?

3.5 Why do you think it is important for learners to be allowed to solve these problems in different ways?

Problem set A	Problem set B
1. Arnold has 17 marbles and loses 11. How many marbles does he have left? 2. Arnold has 17 marbles and Brendon has 11 marbles. How many more marbles does Arnold have? 3. Arnold has 17 marbles and Brendon has 11 marbles. How many more marbles should Brendon get to have just as many marbles as Arnold?	1. Anna has R24. A packet of chips costs R3. How many packets of chips can he buy? 2. Mother has 24 biscuits. She shares these equally among 3 learners. How many biscuits will each child get?

Response to question 3.2

3.2 You will realize that in all likelihood, the approaches used to get the solution differ. Although the problems in Set A can be solved by subtracting 11 from 17, learners do not experience these problems in the same way. Learners might solve the first problem by counting on from 11 or counting back from 17. They might solve the second problem by matching Arnold's and Brendon's marbles on a one for one basis and counting the extra 6 marbles. In terms of the third problem learners would be most inclined to solve the problem by adding/counting on.

Response to question 3.4

The problems in Set B are division problems; they have the structure 24 ÷ 3. However learners do not experience these problems in the same way. In terms of the first problem the child wants to know how many lots of R3 there are in R24. In determining an answer a child might count in 3s: 3; 6; 9; 12; …; 24 keeping track on her fingers and conclude that Arun can buy 8 packets. The structure of the second problem is quite different. The child needs to determine how big each part is if there are 3 equal parts.

Response to question 3.5

We want to develop learners to become skilled mathematical thinkers by allowing them the freedom to select a strategy that they find to be most efficient. Secondly, in the real world, learners encounter problems that are complex, not fixed, not well defined, and lack a clear solution. They need to identify and apply different strategies to solve these problems. Solving problems is different ways allow learners to develop their adaptive reasoning which is one of the strands of mathematical proficiency. However what is important to remember is that problem solving does not happen naturally. They need to be explicitly taught in a way that can be transferred across multiple settings.

4. Study the scenario below and discuss the possible reasons for Nozipho's errors.

Ms Dlamini, a grade two teacher, teaches each number operation separately. She felt that when she asked learners to solve problems that had different operations, it confused learners. The class then had to solve the following problem.

Bongi has 16 marbles. If he wins another 25, how many marbles will he have?

Nozipho did not hesitate, she wrote 16 + 25 = 41. She then looked at her neighbour's work and seeing that he had written 42 she changed her answer to 42. Ignoring this for a moment, we could at this stage (based on her answer) be forgiven for thinking that Nozipho understands what she is doing. Immediately after responding to the first problem, Nozipho was asked to solve the problem:

There are 28 apples. If we put 3 in a packet, how many packets can we fill?

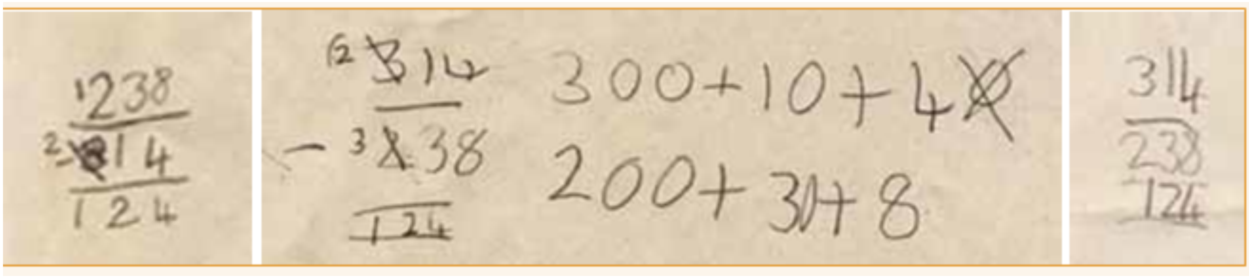
Once again Nozipho hardly hesitated – she wrote: 28 + 3 =31.

Response to question 4

Nozipho, despite being in Grade 2, has already stopped trying to make sense of the situation – she understands her role in mathematics as being to identify the numbers in the problem and to do something with them. It seems that on the day of these problems she regarded "plusing" as being the "thing" to do.

5. Below are examples of learners' responses to the calculation problem: 314 – 238 =

Identify the errors in the three examples and discuss the possible reasons for the learners' errors in the illustrations below



Response to question 5

The errors in the examples of learners' work above could be explained as the result of learners being taught that subtraction involves "taking a smaller number away from a larger number", They were taught the rule that you must always subtract the smaller number form the larger; they have not been taught the conceptual understanding of regrouping and exchange of 10s.

Through completing section F, you would have learnt that:

- it is important for the teacher to understand what learners are thinking as they do problem solving so that the appropriate support can be given to learners.
- learners must be exposed to a range of problems before they can be introduced to the actual operations.
- Learners should be allowed to apply their own strategies.

G. Problem solving using different operation strategies in the Jika iMfundo lesson plans

[You will spend 70 minutes on section G]

In this section, participants examine examples of problem solving exercises from the Jika iMfundo Term 3 lesson plans, answering questions or following instructions set on each. Through this activity, participants will familiarize themselves with the range of problem solving strategies and approaches taught to the learners in each grade.

Activity 8

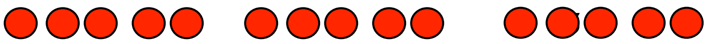
<u>GRADE 1: There are 5 questions to be answered in 20 minutes</u>

Participants work in pairs...

1. Lesson 30: Solve the problem in lesson 30. Then identify the different types of knowledge required by the learners to be able to solve this problem

Lesson 30: Using repeated addition: There are 3 friends. They have 5 sweets each. How many sweets do they have altogether? Draw counters and write a number sentence.

Response to question 1



5 + 5 + 5 = 15

Physical knowledge: The use of counters and pictures
Social knowledge: Teacher tells learners the meaning of altogether (scenario 1 and 2, times 2; remainder); grouping sharing. These are new concepts that must be told and explained to learners. to explain these concepts
Conceptual knowledge is needed for learners to translate the representations to the number sentence i.e. 5 + 5 + 5 =

2. Look at the extract of lessons 31a and 31 b

Together with a partner, role plays these lessons, using the prepared drawings of bicycles and sweets. Then answer the question below:

2a what do you notice about the questions in 31 a and 31 b (*same questions; how and what; build concepts)*
2b. what is the exercise designed to teach the learners? (2 x 3 = 6; 2 x 5 = 10) *repeated addition leads to multiplication; that equal grouping is related to multiplication*)
2c. the teacher could have told the learners that 3 x 2 = 6, 2 x 5 = 10 and drilled them on this. What do you think are the strengths/weakness of that approach compared with the one in the lesson plans?

(*Telling the rule is quick but does lead meaningful understanding; they sequences asked in the LP will help leaners build conceptual knowledge)*
2. *d* How could you support a struggling learner*? (Use concrete aids to do physical grouping and counting in twos)*

Lesson 31a
Show a picture of 3 bicycles each with 2 wheels.

- What do you see? (3 bicycles.)
- How many wheels on each bicycle? (2)
- How many groups of 2 do you see? (3)
- Let's count in twos. (2, 4, 6)
- How many wheels altogether?(6)
- How can I show that as a number sentence? (2 + 2 + 2 = 6)
- How many times 2 do I have
  3 x 2 = 6

Lesson 31b
Show a picture of 5 packets with 2 sweets in each

- What do you see? (5 packets.)
- How many sweets in each packet? (2)
- How many groups of 2 do you see? (5)
- Let's count in twos. (2, 4, 6, 8, 10)
- How many sweets altogether? (10)
- How can I show that as a number sentence? (2+ 2 + 2 + 2 + 2 = 10)
- How many times two do I have
  5x2 = 10

3) **Solve the following examples that show grouping and sharing with remainders. Then state how the example in 35a differs from 35b. Why is it important to vary the examples for learners?**

Lesson 35a
●●●●●●●●
How many groups of 4 can you make? ☐ (2 remainder 0)

●●●●●●●●●●
How many groups of 3 can you make? ☐ (3 remainder 1)

●●●●●●●●●●
How many groups of 4 can you make? ☐ (2 remainder 2)

Lesson 35b Share the balls between the given numbers of friends:

●●●●●●●●●●●
Share 11 balls between 2 friends. Each one gets ☐ balls. (5 remainder 1)

●●●●●●●●●●●●●●●
Share 15 balls between 4 friends. Each one gets ☐ balls. (3 remainder 3)

Response: 35a is about grouping and 35b is about sharing. Both have remainders. Learner need to see the relationship between sharing and grouping. It helps learners see the relationship between multiplication and division.

4. Complete the following problems by using the strategy of drawing. Explain why you think drawing is a recommended strategy in Grade one, but not in higher grades.

4.1 Thompho has 15 flowers. She puts three flowers into each bunch. How many bunches can she make?
(❀❀❀ ❀❀❀ ❀❀❀ ❀❀❀ ❀❀❀ (*5 bunches and no flowers left over)*

4.2 Ntombela has 9 pencil crayons. She packs five crayons into a box. How many boxes will she need?
(✐✐✐✐✐ ✐✐✐✐ (*1 box that will have 5 crayons and another box which will only have 4 crayons)*

4.3 Draw to solve the following problem.
✐✐✐✐✐✐✐✐✐✐✐✐✐✐
Share the pencils between 7 learners. How many pencils will each child get? ☐ (2)

4.4 🕮🕮🕮🕮🕮🕮🕮🕮🕮🕮🕮🕮
Share the books between 3 learners. How many books will each child get?
☐ (4)

Response: Counters and drawings can be used in grade one because the number range is small. As the learners progress to grade 2 and grade 3, they have to find more efficient ways to calculate, rather than using counters and drawings
Consider the number ranges. As the number range gets bigger, children must use more efficient strategies

5. Use counters to solve the problems below. Think about when counters will be used and when drawings will be used.
5.1 Cleo has 20 cookies that must be packed into boxes. 5 cookies go in each box. How many boxes will she need? *(4 boxes)*
5. 2 Tino has 11 marbles that he puts into groups. He puts 3 marbles in each group. How many groups does he have? *(3 groups and 2 remainders)*

Response:
Learners in grade are starting to develop conceptual understanding of concepts and are functioning at a concrete level of cognition, so their conceptual understanding develops by connecting concrete and representational understanding to abstract math process As learners progress to grade 2 and grade 3, they have to find more efficient ways to calculate, rather than using counters and drawings. As the number range gets bigger, children must use more efficient strategies.

<u>GRADE 2: There are 3 broad questions with sub questions to be answered in 25 minutes</u>

Participants work in pairs and practice grade two exercises taken from Jika iMfundo. These exercises focus on strategies using the techniques of doubling, near doubles and breaking down/building up to solve problems

1. Lesson 11 (a, b and c)

Participants will role these exercises with a partner and then answer the questions that follow. The lesson plan uses unifix cubes, but participants can use counters or flard cards.

a. Using technique of doubling

Give each group of learners 75 Unfix blocks.
Ask various learners to make towers of ten blocks each in front of the class.
They are going to take turns to show doubles, using the blocks.
Put down 24 and 24 blocks in groups of 10 and ones. Ask the class: What is the sum? (24 + 24 = 48)
Is there another way to say it? (Yes, double 24 is 48.)
Repeat allowing different learners to participate in the demonstration.
Show 37 and 37 blocks in groups of 10 and ones on the desk. Ask: What is the sum? (37 + 37 = 74)
Is there another way to say it? (Yes. Double 37 is 74.)
Etc. work through more examples of doubles.

b. Using the technique of near doubles

Using the technique of near doubles to calculate:
Use unfixes cubes to solve:
28 + 29 = 57 (Double 28 + 1 = 56 + 1 = 57.)
32 + 33 = 65 (Double 32 + 1 = 64 + 1 = 65)
Practice this example. You can draw representations of base tens
35 + 34 =   45 + 46 =   43 + 45 =

c. Using the techniques of breaking down and building up

Give each group of learners a set of base ten blocks or counters. (If you don't have base ten blocks allow learners to work with flard cards.)
Use base ten blocks/ counters/ flard cards to show how to add the following:
Addition: 23 + 41 = and 45 + 27 =
Subtraction: 55 – 31 = and 61 – 48 =

Calculate using regrouping and exchange strategy. Do not draw pictures show numerical working?
54– 16 =   29 + 37 =   48 – 19 =

Questions related to lesson 11a, b, and c

1. Why do you think it is important for learners to know the facts of doubling of single digit numbers from memory?

Response: It develops factual fluency. They then apply this factual fluency to 2 digit numbers. Learners will then solve number sentences and word problems more quickly with ease. The idea of doubles is that when you have those facts memorized (1+1, 2+2, 3+3, etc.) you then can add problems such as 1+2, 2+3, and 3+4 by just adding one to the sum.

2. Why is doubling 37 more difficult than doubling 34? What knowledge and skills do learners need to double 37?

Response: Doubling 34 is straightforward, and does not require regrouping as in 37. E.g. 30 *doubled is 60 and 7 doubled is 14. This requires regrouping of 10 and 4, the adding 10 to 60 (70) and adding it to 4 (74)*

3. Explain your understanding of "using the technique of near doubles to solve calculations".

Response: This technique is usually used when adding two numbers that are almost the same (One number could be one more or one less). If the smaller of the two numbers are doubled, then the learner has to add 1 to the answer after doubling, if the larger number is doubled, then the learner has to subtract 1. This is called compensation which is a process of manipulating numbers to make it easier to add.
The difference to other strategies is that you remove a specific amount from one number and give t hat amount to the other number. This strategy normally revolves around rounding a number to its n earest multiple of 10 or to align one number with a double fact.

4. Explain how the following sets of calculation in lesson 11c differ:
   - 23 + 41 and 45 + 27
   - 55 – 31 and 61 – 48

   Response: 23 + 41 is straightforward and 45 + 27 requires regrouping because the 5 in 45 added to the 7 in 27 gives you 12, which is a 2digit number which then has to be regrouped.

   55 – 31 is straightforward, but 61 – 48 requires exchange of tens because the 1 in 61 is smaller than the 8 in 48., So 61 has to be regrouped as 50 and 11.

5. **Lesson 24: Solving problems through tables and grids**

Participants will practice these examples and talk about: the value of using tables and grids and how it contributes to learners' understanding of multiplication

a)Heila sells hotdogs at R4 each. Make a table to help her find the amount for large orders.

Number of hotdogs	1	2	3	4	5	6	7	8	9	10
Cost in rands	4	8	12	16	20	24	28	32	36	40

b) Show how to read the table to find out the costs of large orders. For example,
- *If she sells 4 hotdogs, she gets R__?* (R16)
- *If she sells 7 hotdogs, she gets R__?* (R28)
- *If she sells 9 hotdogs, she gets R__?* (R36)

c) Peter babysits. He charges R5 per hour for babysitting. Complete this table for him.

Number of hours	1	2	3	4	5	6	7	8	9	10
Cost in rands	(5)	(10)	(16)	(20)	(25)	(30)	(35)	(40)	(45)	(50)

d) Use a grid to solve the following problem.
27 learners and 1 teacher go on a school trip to a nature reserve. The school pays R1, 20 per person to enter the nature reserve. How much money is paid in total?
- *What is the key word? What is the question? What are the numbers?*
- Draw a picture to show the answer.
- 27 learners + 1 teacher = 28 people (each pay R1,20).
- Draw a table with 7 columns and 4 rows because it will have 28 blocks.

- To find the total amount paid you have to add all the R1, 20 amounts together. (There are different ways this could be done – discuss alternative methods if necessary.)

R1,20	**R1,20**	**R1,20**	**R1,20**	**R1,20**	**R1,20**	**R1,20**
R1,20	**R1,20**	**R1,20**	**R1,20**	**R1,20**	**R1,20**	**R1,20**
R1,20	**R1,20**	**R1,20**	**R1,20**	**R1,20**	**R1,20**	**R1,20**
R1,20	**R1,20**	**R1,20**	**R1,20**	**R1,20**	**R1,20**	**R1,20**

Total: R33, 60 (28 x R1, 20 = R33, 60)

Questions on lesson 24

1. When would you use the strategy of tables and grids to solve problems?

Response: This strategy is most useful when you know the answers to small quantities, but want to calculate larger quantities. Then you will look for patterns. Learners would read the problem, and then look for any numbers, items, or series of events that are repeated throughout that problem.

2. Explain how the strategy of tables and grids leads to learners' understanding of multiplication?

Learners usually discover this strategy when they are learning their multiplication tables. They notice that 2 x 4 is the same as 4 x 2, and so on. They also notice the patterns when they look at a hundreds chart. They can see that one column has all zeros, etc.

3. With regard to the grid, explain alternate ways to calculate all the R1, 20 amounts together.

Learners could first add all the R1, then count in 20s to make groups of R1. Ten add altogether.

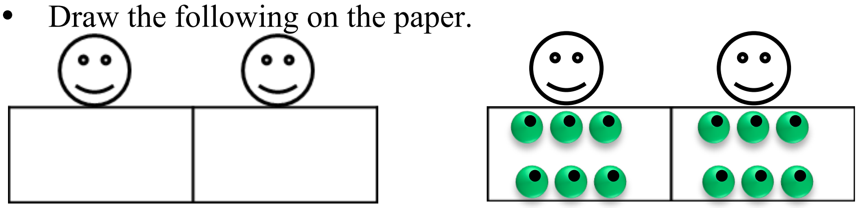
Vertical addition. Using techniques of doubling

4. Problems solving related to fractions.

Participants work in pairs to solve the problem. Participants then answer the questions related to fractions.

Lesson 28: Sharing leading to fractions

i Share 12 counters equally between two learners
- Give each learner a sheet of scrap paper and 12 counters.
- Draw the following on the paper.



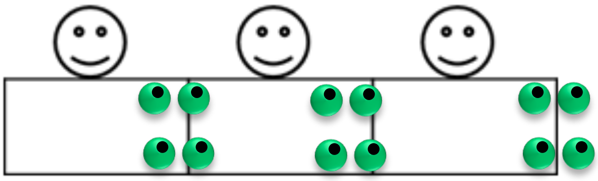
- *How many counters do you each have?* (6)

What is one half of 12? (6)

ii. Do the same with one third and one quarter.
- Learners can draw what they find on scrap paper and you should discuss these drawings

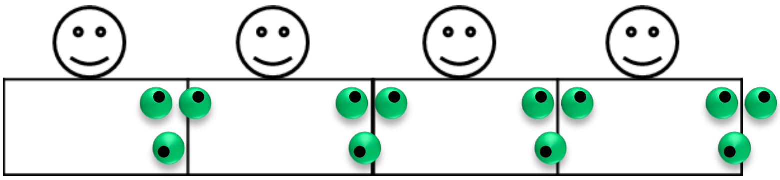
with the whole class.

- Thirds: Share the 12 counters equally between three learners



How many counters did each child get? (4)
What is one third of 12? (4)

- Quarters : Share 12 counters equally among four learners

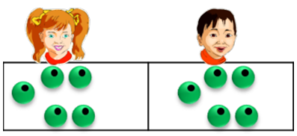


How many counters did each child get? (3)
What is one quarter of 12? (3)

iii. Ask the learners to show you the following:

We are five friends. We have 20 counters. *If we divide them equally, how many counters will each of us get?* (4) *What fraction will each friend get?* (One fifth)

- Look at the picture and answer the questions:
- How many marbles will each child get? (5)
- What fraction of the marbles does each child get? (One half)



iv. Twelve balls divided equally between four friends.

How many balls will each get? (3)
What fraction will each friend get? (One quarter)
Sixteen balls divided equally between two friends.
How many balls will each get? (8)
What fraction will each friend get? (One half)

Lesson 30

i Tell the following "story" to the learners, while drawing the pictures on the board.

- Two friends share three cupcakes equally. 

- *How many cupcakes will each friend get?* (1 and one half cupcakes each.) 

ii Draw the following pictures on the board. Learners need to make up the story and give the answer.

- 

(Four learners share six cupcakes equally. Each child will get 1 and one half cupcakes.)

iii Also do the following: six learners and nine cupcakes. (Each child will get 1 and one half cupcakes.)

Questions related to problem solving fractions

1. Why do you think asking the right questions are important when teaching fractions?

Encouraging the learners to talk about fractions and to use the correct vocabulary will help them understand some of the difficult vocabulary associated with fractions. The questions you use should show the learners how important the correct vocabulary is, so that everyone knows what is being referred to. It's a good idea for the teacher to model some ways of talking about fractions and then drawing the learner's attention to how words are used

2. How do fractions lead to division?

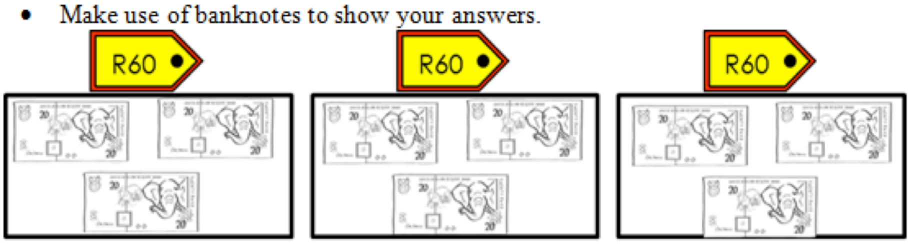
Division is a process of breaking apart whole objects into its fractional parts. When we write a ***fraction****, such as 1/3, the number beneath the line is called the denominator, which is the number into which the whole has been divided*

<u>**GRADE 3: There are five questions with sub questions to be done in 25 minutes**</u>

Participants work with a partner to solve the problems and then answer the related questions

1. Lesson 28: Solve the problem in question (a) below.

a) Linda bought 3 books for R60 each. How much change will she get if she has R200?
   - *What is the key word?*
   - *What is the question?*
   - *What are the numbers?*
- Make use of banknotes to show your answers.



Questions on lesson 28

1. What do you think is the purpose of the three questions included in the lesson 28?

   Response: To check whether learners understand the question. The questions will help learner to identify key information needed to solve the problem.

2. What prior knowledge do you think learner will need to solve the problem?

   This depends on the strategy. If it's number line, know how to make the jumps. If it's through operations, learners will need to know how to recognise notes, calculate money, subtract *money, how to calculate change. How to count on, how to do vertical subtraction*

2. Lesson 7: Using technique of breaking down

Participants solve the following problems using the technique of breaking down

128 + 214 = *100 + 200 + 20 + 10 + 8 + 4* *= 300 + 30 + 12* *= 342* Example with regrouping: 457 + 172 = *400 + 100 + 50 + 70 + 7 + 2* *= 500 + 120 + 9* *= 500 + 100 + 20 + 9* *= 600 + 29* *= 629*	438 – 323 = *400 – 300 + 30 – 20 + 8 – 3* *= 100 + 10 + 5* *= 125* Example with exchange 371 – 265 = *300 – 200 + 70 -60 + 1 - 5* *= 300 – 200 + 70 – 50 + 10 + 1 – 5* *= 100 + 20 + 11 – 6* *= 120 + 5* *= 125*
Jabulile read 425 pages. Buhle read 46 pages. How many pages did Buhle and Jabulile read altogether? (471) Mrs Zuma needs to buy tiles for her bathroom. She needs 178 black tiles and 283 white tiles. How many tiles does she need altogether? (461)	

Question on lesson 7

1. Explain how the concept of "regrouping" differs from "exchange"

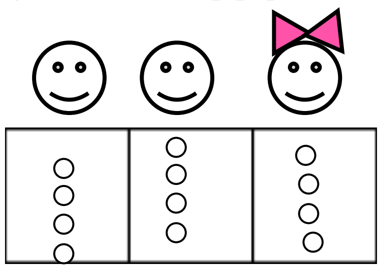
Regrouping is the process of making groups of tens when adding or subtracting two digit numbers (or more) and is another name for carrying and borrowing. Regrouping is when we borrow from one group to give to another so that the operation can be completed. When the unit digit in the first number is smaller than the second number (unit digit) then you need regrouping e.g. 37- 19=
The number is flexibly manipulated to support calculations. Exchange happens when you trade off a ten for ten ones. This usually occurs in subtraction

3. **Sharing leading to fractions**

1. Lesson 31: Share 12 counters among 3 learners

Practice these exercise with your partner. Discuss with your partner the type of questions asked. How do the questions differ from each other? Why are these questions important? Why is it important to give learner opportunities to practice many examples?

i Give learners 12 counters. Tell them to draw faces of three learners (2 boys and 1 girl) and to share the counters one at a time equally amongst the three learners.
They use their Scrap paper/white boards to write on, e.g.



- *How many counters will each child get?* (4)
- *What fraction will the girl get?* (One third since the counters has been shared into 3 groups of equal size.)
- *How many will the girl get? (4)*
- *What fraction did the boys get?* (Two thirds.)
- *How many will the boys get?* (4 + 4 = 8)
- *What is one third of 12?* (4)
- *What is two thirds of 12?* (8)

ii Repeat the above steps, asking the same questions, with the following examples:

- Share 12 counters equally among three boys and one girl (i.e. into quarters – 4 groups of equal size).
- Share 12 counters equally among one boy and one girl (i.e. into halves – 2 groups of equal size.).

iii Draw pictures to calculate.
We are five friends; two boys and three girls. We share 20 counters equally. How many counters will each friend get?

- *What fraction will each friend get?* (1 fifth.)
- *What is one fifth of 20*? (4)
- *What fraction will the boys get?* (2 fifths.)
- *How many counters will the boys get?* (4 + 4 = 8 counters.)
- *What fraction will the girls get?* (3 fifths.)
- *How many counters will the girls get?* (4 + 4 + 4 = 12 counters.)
- *What is three fifths of 20*? (12)
- *What is four fifths of 20*? (16)
- *What is five fifths of 20*? (20)

iv Sharing and grouping

- Give learners counters to help them to work these calculations out practically and cups/containers to hold each person's share. Divide the 9 counters equally between two boys and one girl. Ask:
  - *How many parts will you divide the whole into?* (Three groups – thirds)
  - *How many counters will each child get?* (3 counters in each group – they will each get 3)
  - *What fraction will the girl get?* (One third.)
  - *How many counters will the girl get?* (3)
  - *What fraction will the boys get?* (Two thirds.)
  - *How many counters will the boys get altogether?* (6)

Question on fractions

1. Compare the type of questions asked in grade three to the questions asked in grade two on fractions. How does it show progression in the levels of complexity?

Response: In grade three, the questions are more complex. In grade two, only unitary fractions are taught, but in grade three non-unitary fractions. Therefore the questions assess learners understanding of non-unitary fractions.

In grade two, the focus on fractions is on sharing, with remainders, grade 3 is includes sharing of *fractional parts.*

The questions asked in grade 3 expects learners to respond using the vocabulary related to fractions e.g. One third, one fifth, etc. But in grade 2, the questions ask "how many in one third"

4. Lesson 14: Problem solving related to multiplication and division

Participants complete the exercise on multiplication and answer the questions

a) The use of arrays to solve problems

- Draw an array on the board with 2 columns and 10 rows.
- How many circles are there in each row? (2)
- How many circles are there altogether? Count: (2, 4, 6, 8, 10, 12, 14, 16, 18 and 20)
- Let us write this as an addition number sentence: (2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 20)
- Let us write this as a multiplication number sentence: (2 x 10 = 20 or 10 x 2 = 20)
- The inverse of multiplication is division.
- What would a division number sentence using this array look like?

  (20 ÷ 2 = 10, 20 ÷ 10 = 2)

Here are additional examples of using arrays to solve problems

Draw an array to calculate. I want to make 4 cakes and for every cake I need 3 cups of flour. How many cups of flour do I need? (4 x 3 = 12)

- Draw this as a rectangular array (three by four).
- Write it as an addition number sentence: (3 + 3 + 3 + 3 = 12)
- Write it as a multiplication number sentence: (4 x 3 = c)

Questions on lesson 14

What is your understanding of arrays and what is the purpose of arrays in mathematics?

*Response: An arrangement of objects, pictures, or numbers in columns and rows is called an **array**. Arrays are useful representations of multiplication concepts. Learners can more readily develop an understanding of multiplication concepts if they see visual representations of the computation process. For example, they can picture learners in a marching band arranged in equal rows or chairs set up in rows in an auditorium. These arrangements all have something in common; they are all in rows and columns.*

b) Word problems involving repeated sets.

A vegetable garden has 4 rows of plants. Each row has 2 plants. How many plants are there in the garden?

- Let us write it as an addition number sentence: (2 + 2 + 2 + 2 = 1)
- We can say there are 4 rows with 2 plants in each row. Draw a picture if necessary.
- Let us write it as a multiplication number sentence: (4 x 2 = c)
- Ask learners to make up other stories that lead to multiplication – where repeated sets are involved. E.g. A car can take 5 passengers. How many passengers can 2 cars take? (5 x 2 = 10)

My dad planted 5 fruit trees in a row. He planted 6 rows. Follow the same steps as above

- How many fruit trees did he plant? (30)
- Write it as an addition number sentence: 5 + 5 + 5 + 5 + 5 + 5 = 1 (30)
- Write it as a multiplication number sentence: 6 x 5 = c (30)
- Make up other stories that lead to multiplication where repeated sets are involved. E.g. A car can take 5 passengers. How many passengers can 3 cars take? (5 x 3 = 10)

If I have 42 biscuits and I share them between 3 learners, how many biscuits will each learner get?

- Write this as a number sentence: (42 ÷ 3 = 14)
- Tell another story about the division number sentence. (Mum shares 42 buttons among 3 learners. Each child gets 14 buttons.)
- Do the same for 27, 39, 48, and 54. If you share each of these numbers of biscuits between three learners, how many biscuits will they get (each time)?

Questions on lesson 32

1. Identify the strategies that leads to an understanding of multiplication and the strategies that leads to an understanding of division

Multiplication and division are inverse operations and so the lessons deal with them together. Both of these operations are linked to a basic understanding of grouping. Multiplication is conceptualized in three ways: rectangular arrays, multiplicative comparisons and equivalent groups. Division is conceptualized as grouping and sharing.

As you worked through the activities on problem solving in your group, you would have realized that problems can be solved using different strategies. In the next, section, we will look at how to reflect on our practice when teaching problem solving

H. REFLECTIVE PRACTICE IN THE CONTEXT OF LEARNERS' PROBLEM SOLVING

[30 minutes]

Reflection refers to thinking about what you have done in the classroom, thinking about why you did it, and thinking about if it worked – it is a process of self-observation and self-evaluation. By collecting information about what goes on in our classroom, and by analysing and evaluating this information, we identify and explore our own practices and underlying beliefs. This may then lead to changes and improvements in our teaching especially if we share our reflections with our peers, and together try to solve problems we are facing.

One of the most valuable sources of evidence of 'how well we are doing' is learners' work. Learners 'oral responses and questions in class, and their written work tell us a great deal of what they have learnt. Analysing learners' work, thinking about the reasons for any errors and misconceptions they demonstrate, and then thinking about how we taught the work and how we could have done it better is one way in which we can improve our practice.

Activity 9: Analysing and reflecting on learners' work

The tables below show responses of three grade three learners 'which were scanned from the ANA 2013 diagnostic report. Study the responses carefully and consider the questions which follow:

10.1	Count forwards in 100s	584	585	586	587	588
10.2	Count backwards in 20s	320	310	320	330	240

Complete the table:

10.1	Count forwards in 100s	584	585	586	587	588
10.2	Count backwards in 20s	320	321	322	323	240

Complete the table:

10.1	Count forwards in 100s	584	4	5	8	100
10.2	Count backwards in 20s	320	3	2	0	240

1. What skills and knowledge would learners need to be able to answer this question correctly?

   Response:

   - *Learners need to know the skills and knowledge of counting forwards in 100s and how to count backwards in 20s.*
   - *Understanding the language of "forwards, backwards'.*
   - *Read the instructions.*
   - *Know how to complete the answers in a table format*

2. Read the information below or listen to the facilitator's input about errors and slips. Then answer the questions that follow:

When we talk about work that learners got wrong we use words such as mistake, error or slip. Errors tend to be systematic, persistent and pervasive mistakes performed by learners across a range of contexts. They are based on conceptual misunderstanding. Slips are mistakes that are easily corrected when pointed out. Slips or mistakes are more like careless errors. Since errors are systematic and persistent, they are not necessarily responsive to easy correction or re-explanation of concepts. When doing error analysis, teachers need to examine all learner work – both errors and slips, so that they get an overview of what their learners know/don't know and how to address content issues that is evident in their learners' work. They also will realise how deeply embedded the problem is (error) or not (slip).

2.1 Identify the mistakes made by each learner and state whether it's an error or slip

Response: First learner: The learner counted in 1s instead of 100s. The learner counted in 10s instead of counting in 20s - Error

Second learner: The learner counted in 1s instead of 100s. The learner counted in 1s instead of 20s- Error

Third learner: The learner wrote the digit of numbers in each block- Error

3. What are some of the reasons attributed to learners' errors, slips and misconceptions?

Errors in mathematics may arise for a variety of reasons. They may be due to the pace of work, the slip of a pen, slight lapse of attention, lack of knowledge or a misunderstanding. Some of these errors could be predicted prior to a lesson and tackled at the planning stage to diffuse or un-pick possible misconceptions. In order to do this, the teacher needs to have the knowledge of what the misconception might be, why these errors may have occurred and how to unravel the difficulties for the child to continue learning.

4. Why is it important to think about why errors you observe in learners' work have been made?

Response: Thinking about how errors could be made when answering the question enables the teacher to start to think diagnostically about the way learners think when they go "wrong". This is important because if a teacher can conceptualise how a learner was thinking they will be more able to address the learner's error specifically instead of just re-teaching the content in the same way. It is useful to study learners' working to discover ways in which they have gone wrong.

5. What does this activity (your error analysis) teach you about the way you would reteach the content in the item?

You will go back and reteach the concept to close the gaps that are apparent in the learners' prior knowledge based on what you identified in the learner' response. In the given example, for leaners to complete the pattern in the blocks, the prior knowledge needed is counting in 20s and counting 100s. If you used counting chart to teach counting in 100s and 20s, now you will try another strategy, probably grouping in tens using base ten blocks, counters, etc., or maybe number line strategies. You may even consider teaching the concept using smaller numbers so that learners master the counting strategies with smaller numbers. This means that you will adapt your strategies according to learners needs.

When you are planning to teach this concept again to a new class of learners, you will be mindful of these common errors and teach in a way so that learners do not repeat these errors.

Language could also be a reason for learners' errors. Confusion caused by vocabulary causes a great number of difficulties. Here again demonstrate the meaning to learners, what does forward/ backward means.