



2016 TRAINING WORKSHOP NO.5  
**MATHEMATICS**



**FOUNDATION PHASE**





education

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Department:  
Education

**PROVINCE OF KWAZULU-NATAL**

**Foundation phase  
Just-in-Time Training Workshop 5  
January/February 2016**

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**Facilitator's Guide**

**Maths**



**Jika iMfundo**  
what I do matters

Endorsed by:



Jika iMfundo  
Foundation Phase JIT  
Workshop 5 Mathematics: December 2015  
Workshop guide for facilitators

In this workshop participants will find out more about using problem solving activities in the Jika iMfundo FP Maths materials. They will also find out more about how to teach measurement of area in the Foundation Phase.

Here are some notes that you could use when you facilitate this session. Key points for discussion relating to all of the questions are noted here. There are lots of things that could be said this is just a guideline.

- You should work with your participants guide copy when you facilitate the workshop. This means your page numbers and layout are the same which makes it easier for you to keep in track with the participants. But keep your facilitators guide handy with any special notes you have made to yourself as well should you need to refer to them.
- The participants should work in groups on all of the activity questions.
- You need to circulate and keep group discussion lively and then draw together comments from all of the groups in the last few minutes of each group discussion activity.
- Suggested times are given below – keep a close eye on the clock! If you have more time and want to continue the discussions for longer you are free to do so.

### **Workshop plan**

8.00 – 8.30 – Arrival and distribution of materials for the workshop

8.30 – 10.30 – Session 1: Problem solving (2 hours = 120 min)

10.30-11.00 – Tea

11.00-13.30 – Session 2: Measurement of area in FP (2 ½ hours = 150 min)

### **Activity 1: Problem solving and operations (addition and subtraction) – including 3-digit numbers**

Materials needed for this activity: **Term 4 Lesson plans**

- *Grade 1 Term 4 lessons 9, 10, 11 and 12.*
- *Grade 2 Term 4 lessons 9, 10, 11 and 12.*
- *Grade 3 Term 4 lessons 5, 6 and 7.*

This activity involves sets of questions to guide the discussion for about 120 minutes. Your facilitator will guide you as you break into groups and have large group discussions throughout this time.

## 1. Problem-solving (15 min)

 <p>Reflection (5 min)</p>	<p>Getting yourself to a training workshop in the course of a busy week presents problems that you need to solve. What problems arose for you and how did you resolve them for today's workshop?</p> <p><i>Participants will notes different ideas – circulate and listen to their discussion, noting key points that will reinforce the discussion on maths problem solving that follows this ice-breaker.</i></p> <ul style="list-style-type: none"><li>• <i>Identify key issues to solve. (links to step 1)</i></li><li>• <i>Plan strategy for solution (links to step 2)</i></li><li>• <i>Get going and be sure to arrive on time (links to step 3)</i></li></ul>
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Solving problems is one of the best exercises you can give to your brain. Mathematics is a subject that lends itself to problem-solving activities. We can exercise our own brains (and those of our learners) if we apply our minds to mathematical problems. "Word problems" (often spoken about in maths) are branded by some people as impossible and not worthy of spending time on. People with this attitude are cutting back on the potential impact of mathematics on the development of their learners.

Problem-solving is not an activity which should be reserved for an elite few – we can develop our skills of problem-solving through following guidelines for the structuring of their solutions and through perseverance. We must not expect to solve all problems in a few minutes – this would be unrealistic. Not all problems are so simple! Some do require deep thought and careful consideration in order to be solved. This is where problem-solving trains us for real life, and where our mathematics training can be seen as equipping us for everyday situations and some of the problems we are confronted with in life.

You need to approach a problem systematically. Consider the following steps which could guide you towards successful problem-solving.

### Step 1:

Read the problem carefully and ensure that you understand what the problem is about. Restating the problem in your own words is a good exercise, which will make it clear to you whether or not you have understood the meaning of the problem. It is often a good idea to try and sketch a diagram that assists you to illustrate what is required by the problem.

### Step 2:

Once you have understood what the problem is asking, you have to think of your strategy for solving the problem. Think about what operations you may have to use in the solution. Have you got all the information that you need in order to solve the problem? Have you solved other similar problems which can guide your solution to the current problem? And, can the problem be broken up into smaller parts if it seems too big to solve all at once?

### Step 3:

This step should not present you with large problems if step 2 has prepared you adequately to solve the problem. Here you go about implementing your problem-solving strategy to get to the actual solution to the problem. It is important that you realise the difference between devising a strategy to solve a problem and the actual solution to the problem. Both are important activities. It will become clear to you if you need to change your strategy or find a new one, or if your original strategy was adequate.

#### Step 4:

Once you have solved the problem, a final "logic check" of your solution is never a waste of time. Careless errors can slip into your working (though your strategy may be correct) and lead you to an answer which is not correct. Re-read your work just to be sure that it makes sense and presents a valid, satisfactory solution to the problem. This step of verification may seem like a waste of time, but will often prove its usefulness when on verification; you make small changes and improvements to your answers.

 <p>Reflection (10 min)</p>	<p>How would you summarise each of steps 1 to 4 above in one sentence or phrase?</p> <p><b>Step 1:</b> <i>Read and understand the problem.</i></p> <p><b>Step 2:</b> <i>Plan the solution strategy. Draw on experience. Break it up into smaller sections if this helps.</i></p> <p><b>Step 3:</b> <i>Implement your planned strategy.</i></p> <p><b>Step 4:</b> <i>Check your solution.</i></p>
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#### 2. Types of problems (45 min)

There are different types of problems that we can set. We should try to include a range of problems, rather than set problems that are essentially the same all of the time. Problems will usually have some relevance to real life situations though some can be abstract and call for thought on a more abstract level.

 <p>Reflection (15 min)</p>	<p>Complete and then compare the following two problems:</p> <p>a. Ben has collected 235 cans and Jabu has collected 367. How many more cans has Jabu collected than Ben?</p> <p><math>367 - 235 = 132</math> <i>Jabu has collected 132 cans more than Ben.</i></p> <p>b. The number 400 is 250 bigger than another number. What number is that?</p> <p><math>400 - 250 = 150</math> <i>400 is 250 bigger than 150.</i></p> <p>c. In what way are these problems similar/different?</p> <p><i>They both involve subtracting to find the answer. The first one is contextualised, the second one is abstract. They both have one correct answer.</i></p>
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Many problems set in maths have a single solution. As a real life model, this is not adequate, since many real life problems have more than one solution depending on varying circumstances. We need to try and include some multiple solution problems in our range of problems set. The same applies

to ambiguity in problems: not all problems should be simple and straightforward, as this does not equip our learners for the ambiguities that arise in real life.

 <p>Reflection (15 min)</p>	<p>Complete and compare the following two problems:</p> <p>a. Ben has collected 145 cans and Jabu has collected 178. How many cans have they collected altogether?</p> <p><math>145 + 178 = 323</math> <i>Ben and Jabu have collected 323 cans altogether.</i></p> <p>b. Class A and Class B are competing to collect the most cans.</p> <p>i. If Class A collects 568 cans, how many cans should Class B collect to win the competition? <i>569 cans (or more).</i></p> <p>ii. Which class will win the can collection competition? Discuss your answers in your group. <i>Either class could win – it will depend on how many cans Class B collects. If Class B collects less than 568 cans then Class A wins, if Class B collects more than 568 cans, then they will win.</i></p> <p>c. In what way are these problems similar/different?</p> <p><i>Both of the problems relate to numbers of cans collected and thinking about sums and/or differences between these numbers. BUT problem b) has more than one correct solution to both parts of the question while problem a) has only one correct solution.</i></p>
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Problems need to be graded and we need to include problems from the most simple to those which are more complex and difficult, if we are to give our learners experience in solving the range of problems which they might encounter in real life. As we said earlier, you need to be confident about problem-solving yourself.

Here are some problems for you to try. The very best way to improve your own problem-solving skill is to put it to the test. Remember that advice from friends on the solutions can be useful, but don't rely too much on others – exercise your own brain as much as possible! Look back at the guidelines for problem-solving strategies at this stage – they might be helpful. (Note that the problems set in the following activity are mostly at a Grade 3 level, so that operations with 3-digit numbers can be revised while you are in this discussion group.)



Activity  
(15 min)

1. The distance from Johannesburg to Durban is approximately 600 km. If I have done 486 km of the journey, about how far do I still have to go?

$$600 - 486 = 114$$

*I have about 114 km still to go.*

2. There were 563 learners in the school hall for assembly. The juniors (297) went back to class early. How many seniors were left in the hall?

$$563 - 297 = 266$$

*There were 266 seniors left in the hall.*

3. Mbongeni is mad about jigsaw puzzles. He has one puzzle with 100 pieces, one with 250 pieces and a real giant with 500 pieces. How many puzzle pieces are there in these three puzzles altogether?

$$100 + 250 + 500 = 850$$

*There are 850 puzzle pieces altogether*

4. Sipiwe and Rose had to fold serviettes for their older sister's wedding. Their little brother ran past with a jug of water and fell, spilling water on some of the 232 serviettes that they had folded. They quickly sorted them, but still had to set aside 87 serviettes. How many folded serviettes do they still have that they can use for the wedding celebration?

$$232 - 87 = 145$$

*There are still 145 that they can use.*

### 3. Problem solving using different operation strategies in the Jika iMfundo lesson plans (60 min)

There are different ways in which you can do the calculations when you solve problems. As a teacher you should feel confident to do them in any one of these ways, but you should allow your learners some flexibility and mark all correct working in the appropriate way.

 <p>Activity</p> <p><i>Pace yourself:</i></p> <p>Gr 1 (20 min)</p> <p>Gr 2 (20 min)</p> <p>Gr 3 (20 min)</p>	<p>Refer to the Jika iMfundo lesson plans to look at different suggested ways of doing working when doing addition and subtraction.</p> <p>Work through each of these lessons in your group, noting the strategies suggested for doing the solutions.</p> <ul style="list-style-type: none"><li>• <i>Grade 1 Term 4 lessons 9, 10, 11 and 12.</i></li><li>• <i>Grade 2 Term 4 lessons 9, 10, 11 and 12.</i></li><li>• <i>Grade 3 Term 4 lessons 5, 6 and 7.</i></li></ul> <p>Make notes in your lesson plans and also in this space of ideas and questions that you have.</p> <p><i>Circulate and share discussions with the groups. Make sure you talk about the alternative algorithms suggested in the lessons for the calculations.</i></p> <p>Solutions to additional calculations for consolidation. Please note that the calculations below are one suggestion for each question. Other possible horizontal workings are possible. Check that correct mathematical steps have been followed and accept alternatives.</p> <p>a. <math>100 - 78</math> <math>= 90 + 10 - (70 + 8) = 90 - 70 + (10 - 8) = 20 + 2 = 22</math></p> <p>b. <math>100 - 36 =</math> <math>= 90 + 10 - (30 + 6) = 90 - 30 + (10 - 6) = 60 + 4 = 65</math></p> <p>c. <math>300 - 123</math> <math>= 200 + 90 + 10 - (100 + 20 + 3)</math> <math>= 200 - 100 + (90 - 20) + (10 - 3) = 100 + 70 + 7</math> <math>= 177</math></p> <p>d. <math>400 - 276</math> <math>= 300 + 90 + 10 - (200 + 70 + 6)</math> <math>= 300 - 200 + (90 - 70) + (10 - 6) = 100 + 20 + 4</math> <math>= 124</math></p> <p>e. <math>324 - 278</math> <math>= 324 - (200 + 78)</math> <math>= 324 - 200 - 78 = 124 - 78 = 100 + 24 - 78</math> <math>= 100 + 10 + 14 - (70 + 8) = 110 - 70 + (14 - 8)</math> <math>= 40 + 6 = 46</math></p> <p><i>You can make up many more similar problems for yourself and your learners if you want to have more practice.</i></p>
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#### 4. Reflection – the importance of problem solving (AT HOME)

In this session you have thought about problem solving and different kinds of problems. In your classroom you will have to develop the problem-solving ability of your learners. To do so, you will have to ensure that you set them sufficient challenging problems and insist on the learners themselves solving these problems. **DO NOT DO THE WORK FOR YOUR LEARNERS.** They need to struggle through problems (you should guide and assist them) in order to build their own problem solving skills.

You will also have to ensure that your class is divided into functional groups, where meaningful interaction occurs. Problem-solving can (and should) be an interactive activity: do not leave the learners on their own to solve all the problems you set them; let them work in groups (pairs, fours, etc., depending on the nature of the problem and the size of the class).

Ultimately learners do need to be able to solve problems independently, but the preparation they get working in groups with peers and as a whole class will help them to find this activity less daunting.

 <p>Reflection</p>	<p>Why do you think problem-solving is such a good activity in a maths classroom?</p> <p><i>Many different ideas will come to mind, such as:</i></p> <ul style="list-style-type: none"><li>• <i>Develops independent thinking skills.</i></li><li>• <i>Develops interpretive skills.</i></li><li>• <i>Gives opportunities for application of maths algorithms.</i></li></ul> <p>Why do you think group work is valuable in a problem-solving context?</p> <p><i>Peer learning possibilities.</i></p> <p>How will you monitor the pace of a lesson when your learners are doing problem solving?</p> <p><i>Keep an eye on the time – both in the lesson and in regard to the week's lesson plans. Don't allow things to drag on too long, but give as much time as you can to allow for meaningful activity.</i></p>
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## Activity 2: Teaching measurement of area in the Foundation Phase

Materials needed for this activity: **Term 4 Grade 3 Lesson plans, Maths Dictionary, scissors)**

- *Grade 3 Term 4 lesson 12 and 13.*

In this discussion you will refer to the *Lesson Plans* but you will also do hands-on activities related to the teaching of measurement in the FP. These will give you experience on how to work with the lesson plan activities relating to the teaching of place value. You will make your own manipulatives using the cut-outs in the attached hand-out.

This activity involves sets of questions to guide the discussion for about 150 minutes. Your facilitator will guide you as you break into groups and have large group discussions throughout this time.

### 1. Introduction to concepts and relevant ideas (20 min)

 Reflection (5 min)	What is a concept?  <i>Definitions may vary. A concept is an idea (abstract).</i>
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Children are aware of physical objects and their characteristics **before** they develop a concept of number and measurement. We must ensure that they **fully understand** the concepts (of length or area for instance) of the things that we measure **before** we teach them how these are measured. This is because the way in which we assign numeric values to quantities is by comparing them to other quantities similar to themselves.

Some of the characteristics of physical objects have **size** or **amount**. We will call these physical quantities. Ultimately it is the physical quantities that we measure. This conceptual understanding will help learners to use the correct language when they talk about measurement and use units of measurement with greater confidence.

Think of the learners in your class.

- How many of them are there?
- They have physical characteristics such as height and personality; they are made of substance, they take up space, they have eye colour, hair, a smile, length of hair, academic ability, artistic ability, sporting ability, shoe size; and so on ...
- Some of these characteristics have size or amount – we call these physical quantities: mass, volume, length.
- Ultimately we will be able to measure the characteristics which have “size”.

 Reflection (10 min)	<b>Remember this: we measure physical quantities not physical objects!</b> Based on the notes above – what does this statement mean to you? <i>We can only measure physical characteristics which have “size”</i>  What physical characteristics of yourself could you measure?  <i>Several – for example height, mass, waistline, hand-span, etc.</i>
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Measurement involves quantifying (giving a number to) physical characteristics. When we quantify we assign a numeric value to something. For example, we can say that a belt is 90 cm long, or a cup holds 250 ml, or the mass of the child is 34 kg. We cannot quantify things to which we cannot assign a numeric value. For example, if the belt is black, we cannot say how black it is by giving a number ... black.

 Reflection (5 min)	<p>Think about physical characteristics such as "length" and "surface".</p> <p>How do these show themselves?</p> <p><i>Length – the idea of “how long” or “how short”.</i></p> <p><i>Area – amount of surface covered.</i></p> <p>How do we measure them?</p> <p><i>By comparing them to the length/area of a non-standard or standard unit.</i></p>
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## 2. The language of measurement (60 min)

We need to take care with the language that we use in giving instructions and in phrasing questions about measurement. We cannot say: "measure that boy" or "measure that table" – we must specify **which** quantity relating to the boy must be measured. We could say "measure the height, mass, waistline of the boy", or we could say "measure the height, width, length of the table".

Here then are some key questions relating to our teaching of size and measurement:

### 2.1 What do we measure? (20 min)

We measure the size (amount) of a physical quantity pertaining to a physical object. Length, for instance, may be great or small. We measure the length of an edge, not the edge itself. We may measure the mass of a ball. Then we would say "the mass of the ball = 3 kg". (We DO NOT say the ball = 3 kg.)

This shows the need for careful use of **language** in this topic of measurement, if we are to avoid speaking unclearly or ambiguously. We must say exactly what we mean, and give clear instructions to our learners, so that they will know to which quantities we are referring. We must not allow any confusion between a thing itself and its quantifiable characteristics.

 Reflection (15 min)	<p>Why is it not correct to say “the table = <math>5m^2</math>”</p> <p><i>Because the area of the table is <math>5m^2</math>, not the table itself. The table is an object.</i></p> <p>Write out a few clear instructions to learners, calling on them to measure some different physical quantities in relation to different objects (say what you would expect their answers to be). Take careful note of how you use language to do this.</p> <p><i>Suggestions will vary. Work on the different ideas making sure correct language is used at all times. Allow everyone a chance to give an instruction and state a measurement. For example:</i></p> <ul style="list-style-type: none"> <li><i>What is my mass? My mass is 67 kg.</i></li> </ul>
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## 2.2 What are non-standard and standard units? (20 min)

We choose suitable units to measure with. The units must possess the property of that which we are trying to measure.

For example, to measure the length of the edge of a desk we could use a pencil. Then a pencil = 1 unit, and the length of the desk would be ... units. The pencil is then our chosen standard, its size taken to be 1. Such a unit is called an arbitrary standard, or a *non-standard unit*. Clearly, problems could arise if such a unit were used to measure formally. But these non-standard units help learners to familiarise themselves with measurement concepts because they embody the concept themselves.

There are certain accepted *standard* units used for measuring all of the physical quantities (for example, *cm* and *mm*). These are part of what we will teach when we teach about measurement. We teach about the standard units after learners have understood the necessary concepts.



Activity  
(15 min)

1. Name some non-standard units for length.

*Answers will vary – e.g. a pencil, a pen, a book, a hand, etc.*

2. How do we use these units?

*We use them to mark off how many long the object we are measuring measures. E.g. the desk is 3 pencils wide.*

3. Name some non-standard units for area.

*Answers will vary – e.g. a block, a book, etc.*

4. How do we use these units?

*We use them to mark off how many of area the object we are measuring covers. E.g. the desk is 6 books in area.*

## 2.3 What is measuring? (20 min)

Measuring is the process whereby we assign a number to a physical quantity by comparing it with a standard physical quantity, whose size we arbitrarily decide shall be the unit size (i.e. its size is taken to be one). In our teaching of measurement, we will use non-standard units to assist the formation of the concept we are teaching before we go on to teach the accepted standard units applicable to that which is being measured. It is vital that you understand the difference between non-standard units (used in concept development, such as the pencil, and your other examples, mentioned above) and internationally accepted standard units (such as millimetres, centimetres and so on). You will use both in your teaching of size.

When we measure, we need to realise that there is a difference between **pure numbers** and **denominate numbers**.

Pure numbers relate simply to the concept of number, of "how much", without concerning themselves with "of what". Denominate numbers specify what they are counting. "5" is a pure number, whereas "5 dogs" is a denominate number (dogs is the denomination). This is an important distinction for you to remember in the teaching of size, since all measurements will be denominate numbers.

You must teach your students to record their measurements correctly, giving the unit of measurement each time. The denominate numbers (units) need to be properly used when it comes to computations involving these numbers, which we do teach, once the concepts and measuring skills have been taught.

 Reflection (15 min)	<p>Why are all measurements given as denominate numbers?  <i>Because measurements are given with units – they are not pure numbers.</i></p> <p>Give an example of a denominate number using the concept of area.  <i>The area of the table is 5m<sup>2</sup>.</i></p> <p>Why should learners remember to put in the unit when they give a measurement?  <i>It is the correct way to give a measurement.</i></p>
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### 3. Conservation of size – some practical exercises (25 min)

#### 3.1 Piaget's conservation tests

The psychologist Piaget is well known with respect to his ideas on conservation of number. In relation to measurement, Piaget's ideas can be used to check our learners' readiness to proceed with the measurement of things (such as area). We need to check that the learners have achieved conservation of the concept before we can teach about how to measure it. Conservation of the concept means that they have a clear understanding of the constancy or unchanging nature of that concept (for example area). Piaget went further to say that if a learner is able to explain that distorted amounts could be restored to their original appearance, the learner has achieved the concept of reversibility.

We will consider the conservation test for area (but the same concept does apply to mass, volume, length and so on). Different children develop at different paces, and we cannot assume that "all 12 year olds" should have achieved conservation of the size concepts. It does not take very long to test for the conservation of these concepts, so we should always just take that little extra step to check for conservation before we proceed to teach the measurement of different amounts.

 Reflection (10 min)	<p>Why is it important that a child achieves conservation of area (for instance) before we teach the child to measure area?</p> <p><i>Conceptual understanding must be achieved before we can work with the concept.</i></p> <p>What is the difference between conservation and reversibility, using the terms as Piaget spoke about them?  <i>Conservation of the concept means that a clear understanding of the constancy or unchanging nature of that concept (for example area) has been reached.</i>  <i>Reversibility means that the learner is able to explain that distorted amounts could be restored to their original appearance.</i></p>
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#### 3.2 Conservation of area

Area is the amount of surface covered by a shape. To test for conservation of area, show the learner two postcards which are exactly the same. They have the same area. Let her satisfy herself that they have the same area.



Now take one of the postcards and cut it into two parts (second display). Ask the learner if the two areas covered are still the same, or if they cover different areas. You could then further distort the one postcard by cutting it up into a few pieces (third display). Then ask again if the two displays still cover the same area.



Initial display

Second display

Third display



Activity  
(15 min)

1. Make your own displays using the given cards.
2. What will you ask the child as you show her the displays?  
*Are the areas covered in the different displays the same, or do they cover different areas?  
If they are different, how are they different?*
3. How will you assess her responses in relation to conservation of area?  
*If the learner is able to see that area is not changes when it is changed in a display, then conservation of area has been achieved. If the learner believes that the area has changed (for example become greater) when the display looks "bigger" then conservation of area has not been achieved.*
4. How will you know if she has achieved reversibility of the concept of area?  
*Reversibility means that the learner is able to explain that distorted amounts could be restored to their original appearance.*
5. What other apparatus would be useful in tests for conservation of area?  
*Magazine pages, post cards, etc.*

#### 4. Measurement of area in the Jika iMfundo lesson plans (35 min)

Refer to the Jika iMfundo lesson plans to see how the lessons on area have been planned. Work through the lesson in your group, noting the strategies suggested for doing the solutions.

 <p>Activity (25 min)</p>	<p>In this activity you will work through the following lessons:</p> <ul style="list-style-type: none"><li>• <i>Grade 3 Term 4 lesson 13 and 14.</i></li></ul> <p>Prepare the lesson materials using the printables attached:</p> <ul style="list-style-type: none"><li>• <i>Squares template; rectangular shapes.</i></li></ul> <p>Discuss in your group how the lessons incorporate the theoretical ideas discussed in this session.</p> <p>Make notes in your lesson plans and also in this space of ideas and questions that you have.</p> <p><i>Circulate and share discussions with the groups. Make sure that participants have cut out their square blocks and are using them correctly to measure the area of the given shapes. Also check that they know how to count the number of units (squares, rectangles, triangles, etc.) when they use non-standard units to measure area.</i></p>
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Acknowledgement: The following resource was used in the preparation of this workshop. Sapire, I. (2010). *Mathematics for Primary School Teachers*. Saide and the Wits School of Education, University of the Witwatersrand, Johannesburg.