PARTICIPANT'S HANDOUT

2020 TRAINING WORKSHOP NO.1

MATHEMATICS

GRADES 10-12

PILO
IMPROVING LEARNER PERFORMANCE

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

FET
Just-in-Time Training Workshop
2020: No. 1

Participants' Handout

Mathematics

Jika iMfundo
what I do matters

Endorsed by:

Improving Learning Outcomes in Mathematics

Workshop Aim

The aim of this workshop is to provide you with an opportunity to explore the following:

- A discussion about the new format of the Planners/Trackers for 2020.
- Theory relating to the development of Mathematical understanding.
- The use of Polya's problem-solving technique in Paper 1 questions in the FET phase.
- The use of Polya's problem-solving technique in Paper 2 questions in the FET phase.
- The need for more problem-solving in our classrooms and the ways in which this can be accomplished.

Jika iMfundo Workshop Programme		
	Description	**Resources**
Session 1 20 min	**Introduction and Welcome**	
20 min	Pre-workshop activity	
Session 2 30 min	**A discussion about the new format**	
30 min	Activity 1: A discussion about the format of the Planners/Trackers for 2020	Flip chart paper Markers
Session 3 30 min	**How to solve it**	
30 min	"How to solve it" by G Polya -some aspects about how to solve problems Activity 2: A related discussion	Power point presentation
Session 4 75 min	**Using Polya's problem-solving technique in Paper 1 questions**	
60 min	Activity 3: Do provided Algebra and Calculus questions using Polya's technique.	Flip chart paper Markers
15 min	Activity 4: Different groups share their solutions.	Flip chart paper Markers
Session 5 75 min	**Using Polya's problem-solving technique in Paper 2 questions**	
60 min	Activity 5: Do provided Trigonometry, Analytical & Euclidean Geometry and Statistics questions using Polya's technique.	Flip chart paper Markers
15 min	Activity 6: Different groups share their solutions.	Flip chart paper Markers
Session 6 30 min	**The problems with problem-solving at school**	
15 min	Activity 7: List the issues you face with problem-solving in your classroom. With each issue, try to suggest a solution.	Flip chart paper Markers

15 min	Activity 8: Feedback from groups	Flip chart paper Markers
Session 7 15 min	**What is the solution? Why do we need more problem-solving in our classrooms?**	
15 min	Activity 9: General discussion: How can we bring problem-solving into our classroom on a regular basis, given the curriculum time constraints?	Flip chart paper Markers
Session 8 25 min	**Closing**	
20 min	Activity 10: Post-workshop activity	
5 min	Evaluation of this workshop	

Session 1: Introduction, welcome and pre-workshop activity 25 min

The pre-workshop 20-minute activity consists of a short assessment, which will assess your ability to solve problems under the pressure of time constraints.

Session 2: The new format of the planners/trackers 30 min

The planner and tracker you are familiar with has been changed in various ways. You should refer to the planner you have brought with you or the exemplar handout out as you engage with the general orientation to it and then do Activity 1.

Note: The exemplar Grade 10 planner does not have all the contents of the planner. Most of the planning tables and assessment exemplars have been left. The contents list makes clear everything that is included and it does have sufficient material for the purposes of this workshop.

General orientation

- All the tracking columns have been removed.
- Two terms' planners are now in one document; two books of planners will thus be distributed in the year.
- Teachers must keep these documents – they will not get a new version each year.
- Tracking will be done on a separate sheet – a copy is included for photocopying.

- The changes have been made to save costs, to simplify distribution and also to provide a tracker for teachers to use with any planning tool they choose – the ATP or their own work schedules.
- Everything that was in the previous planners and trackers has been included – but some resources, that were repeated each term, are now only given once., with some changes to give term details in one resource rather than two.
- The tracking template (see annotated version in the introductory section) now encourages:
    - daily as well as weekly reflection
    - teachers to think about curriculum coverage in terms of what learners have learnt, as well as what they have covered - this is done by asking them to think about the number of leaners they feel are working at a level 4 level of

Remember: Jika iMfundo is a campaign to improve learning outcomes. Its main strategy is to strengthen the practices that support curriculum coverage. Look at the cycle of these practices shown on the first page of the revised planner and read the brief notes about each one.

Activity 1

Work with a partner and together answer the questions below.

After about 30 minutes, the facilitator will ask for pairs to report back for more general discussion.

1. What is meant by 'covering the curriculum'?
2. Why is improving curriculum coverage in our schools so important?
3. How do the following teacher activities support improved curriculum coverage?
    a. Planning what to teach each day of the term
    b. Reflecting on the day's teaching
    c. Tracking curriculum actually covered against what was planned
    d. Reporting on curriculum coverage to the departmental head
    e. Sharing curriculum coverage challenges with peers; the DH and advisers

4. What is the role of the DH in supporting improved curriculum coverage- what must he/she do?
5. What sort of attitudes and ways of interacting must exist among members of a department if curriculum coverage is to be improved?
6. How do the resources in the planner support improved curriculum coverage – think about:
    a. The planners themselves
    b. The assessment plans/programme
    c. Exemplar assessment tasks with memo and analyses of cognitive levels
    d. Resources related to content and pedagogy
7. What is your general view of the revised version – in what ways is it or is it not an improvement on the previous version?

Session 3: How to solve it 30 min

George Polya wrote a book called "HOW TO SOLVE IT– A new aspect of Mathematical Method". His first copy was printed in 1945. His approach to problem-solving is timeless.

Below are some excerpts from his book, followed by a diagrammatic representation of his 4 problem solving steps

Your facilitator will review these with you.

Some excerpts:

- A teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve problems with stimulating questions, he may give them a taste for, and some means of, independent thinking. (Preface, p. v)

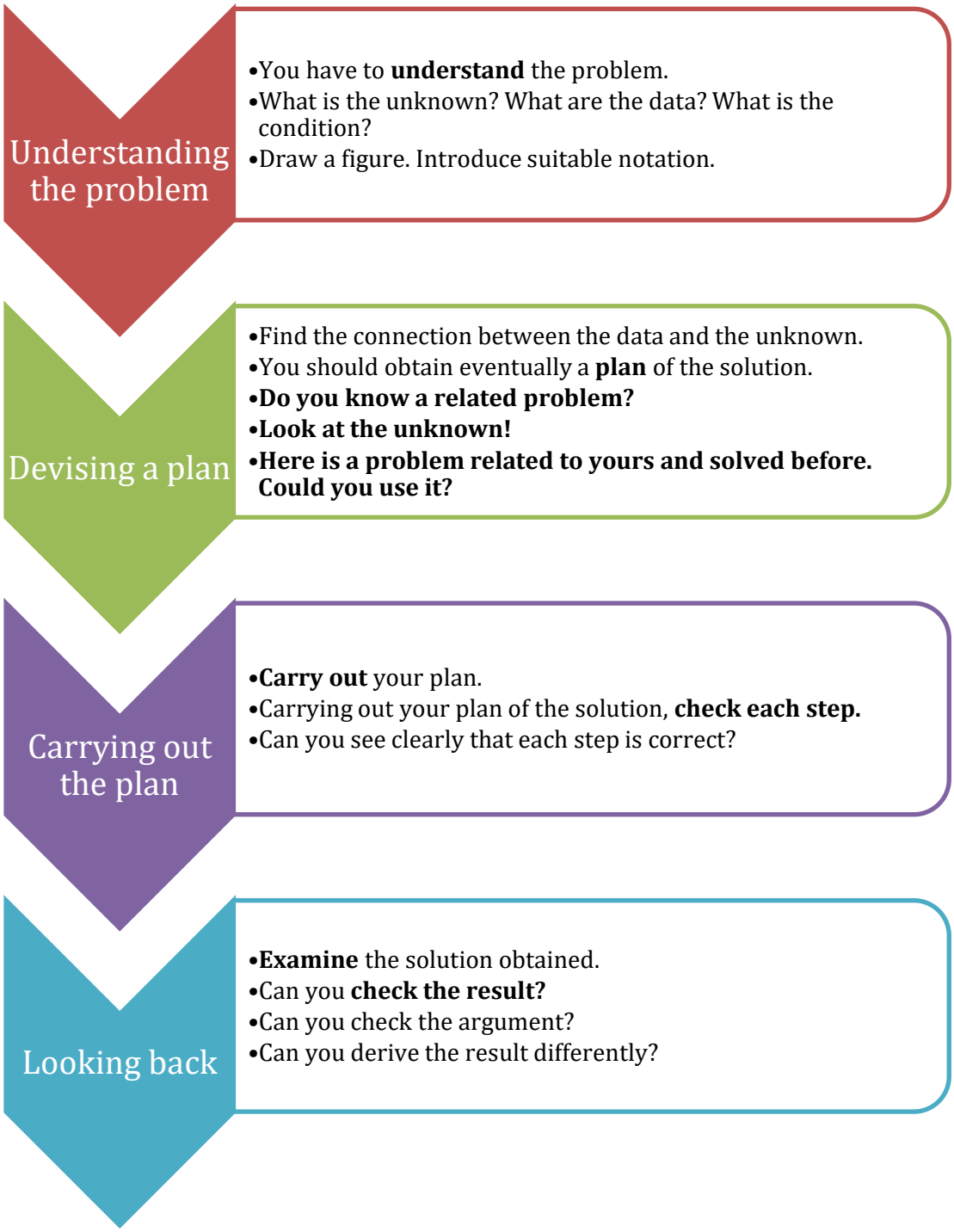
- One of the most important tasks of the teacher is to help his students. The student should acquire as much experience of independent work as possible. But if he is left along with his problem without any help or with insufficient help, he may make no progress at all. If the teacher helps too much, nothing is left to the student. (p. 1)

- Solving problems is a practical skill like, let us say, swimming. We acquire practical skill by imitation and practice. Trying to swim, you imitate what other people do with their hands and feet to keep their heads above water, and, finally, you learn to swim by practice swimming. Trying to solve problems, you have to observe and to imitate what other people do when solving problems and, finally, you learn to do problems by doing them. (p. 4 & 5)

- The teacher who wishes to develop his student's ability to do problems must instill some interest for problems into their minds and give them plenty of opportunity for imitation and practice. (p. 5)

- When the teacher solves a problem before the class, he should dramatize his ideas a little and he should put to himself the same questions which he uses when helping the students. (p. 5)

This is a list of 4 main steps which Polya provides for solving problems:
(A more comprehensive version is provided in **Appendix 1** of your Resources handout).



Activity 2: Group discussion

1. What constitutes a *problem* in mathematics? When you consider the notion of problem-solving, do you automatically think of a "speed, distance, time" 'word' problem which is common in the Grade 11 curriculum?
2. Is there a step in Polya's list which you would like to modify? Is there anything which you would like to add?
3. Would anyone like to provide an example of a type of a mathematical problem which we could solve, using the above list?

You can download a free **complete** copy of *How to Solve It* by George Polya on https://lms.umb.sk>content (with a new foreword by John H. Conway).

Session 4: Using Polya's problem-solving technique in Paper 1 questions 75 min

Activity 3 **60 min**

We are going to solve the following problems, selected from Grade 11 and 12, using Polya's technique which we discussed above. Use Appendix 1 of your Resources handout to familiarise yourself with the details of the method.

The mark allocations have been left out of these questions, so that they do not influence your thoughts!

QUESTION 1:

Given that $P = \frac{5^x+5^x+5^x+5^x+5^x}{5^{x+5}}$, determine the value of P.

QUESTION 2:

The straight line with equation $y = mx + 32$ is a tangent to the graph of $f(x) = -2x^2 - 4x + 30$. Calculate the possible value(s) of m.

QUESTION 3:

A group of Grade 11 learners goes out for lunch to celebrate the start of the holidays. They agree to share the expenses equally. The total bill is R1200. Ten learners do not have any money, so the other learners pay an extra R20 each. Determine the number of learners in the group.

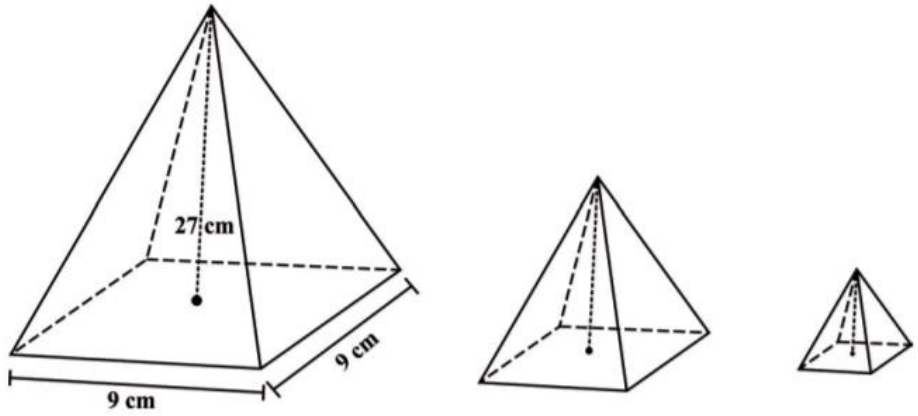
QUESTION 4:

The equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have real and equal roots. Solve for a and b, where $a > 0$ and $b > 0$.

QUESTION 5:

A solid right pyramid with a square base has a perpendicular height of 27 cm. The base has a length of 9 cm. This pyramid is replicated under the following constraints:

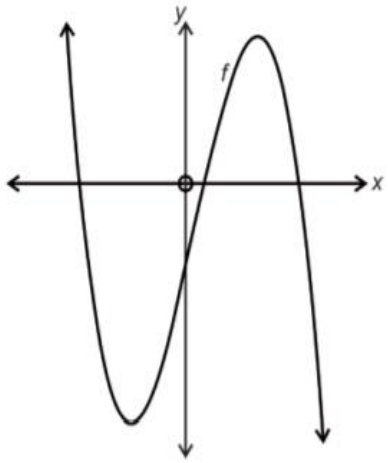
The base area and perpendicular height of each replica is one third of the previous one.



Determine the total volume of all the pyramids replicated, if this replication continues indefinitely.

QUESTION 6:

The graph of a cubic function f is sketched below.



The equation of the curve is given as $f(x) = -x^3 + bx^2 + cx - 3$, where b and c are constants.

$f(1) = 4$ and $f''\left(\frac{1}{2}\right) = 1$

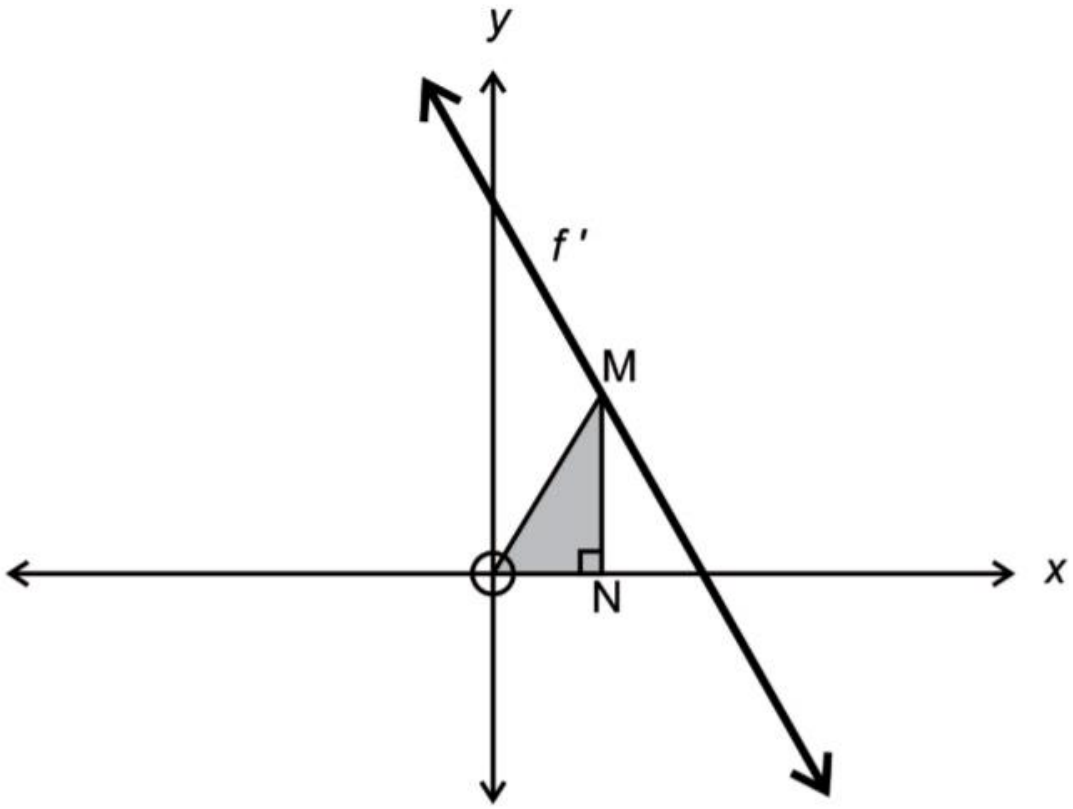
(a) Determine the values of b and c.

(b) Determine the values of x for which the graph is concave up.

QUESTION 7:

In the diagram, the vertices of the shaded right-angled triangle OMN are O $(0;0)$, the variable point N $(x_1;0)$ which is on the x-axis where $0 \le x_1 \le 3$ and point M which lies on the line $2y + 3x - 6 = 0$.

The line represents the graph of the first derivative function of the function of f.



The area of the shaded region is given as $A = rx^2 + tx$ and $f(x) = rx^2 + bx + c$ has a stationary point at $(x;5)$.

Determine whether the value of x_1 that yields the maximum area of Δ OMN is also the value of x_2 that yields the maximum distance between the graphs of f and its derivative f'.

Show all working.

Activity 4 **15 min**

Now we can share your solutions and discuss aspects of them in relation to the steps involved in Polya's problem-solving technique.

(Solutions to the above exam questions can be found in **Appendix 4** in the Resources handout.)

Session 5: Using Polya's problem-solving technique in Paper 2 questions 75 min

Activity 5 **60 min**

We are now going to repeat Activity 3, but the questions have been selected from Trig, Analytical & Euclidean Geometry as well as from Statistics.
The mark allocations have been inserted for this activity.

QUESTION 8:

If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°;270°]$, calculate, **without using a calculator**, the values in simplest form of:

5.1.1 $\sin 2\theta$ (4)

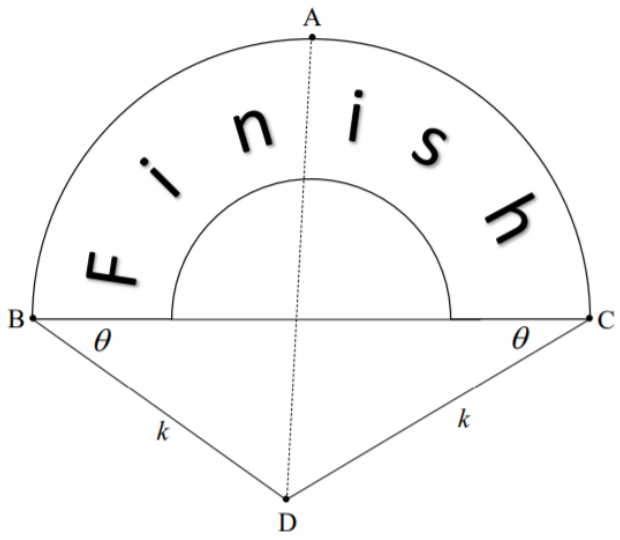
5.1.2 $\sin^2 \theta$ (3)

QUESTION 9:

At the end of a marathon there is a semi-circular banner with the word 'Finish' written on it. It stands perpendicular to the horizontal ground.

In the diagram below:

- B, C and D are points on the horizontal ground.
- The highest point A on the vertical banner is connected to D by a straight rope AD.
- $C\hat{B}D = B\hat{C}D = \theta$.
- $DC = DB = k$.



(a) Show that $BC = 2k\cos\theta$. (4)

(b) Prove that $AD = k$. (6)

QUESTION 10:

Prove that the radius of the circle having the equation

$$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 3 = 0$$

can never exceed $\sqrt{13}$ for any value of θ. [6]

Show all your calculations.

QUESTION 11:

(a) The following information is given about the test results of a class:

$\sum_{k=1}^{20}(x_k - \bar{x})^2 = 156$ and $\sum_{k=1}^{20} x_k = 1220$

Determine

(i) The number of pupils in the class (1)

(ii) The mean mark (1)

(iii) The standard deviation (2)

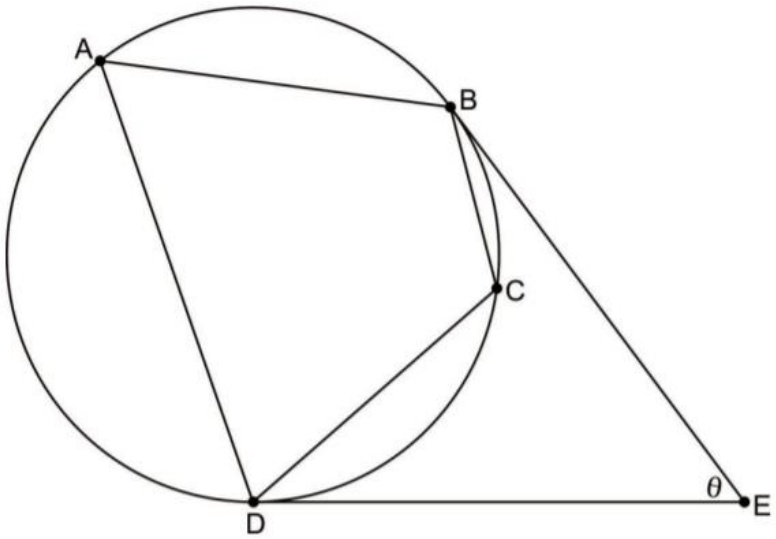
(b) In a certain data set the interval that is one standard deviation from the mean is [350;380]

Calculate the mean and the standard deviation for this data. (4)

QUESTION 12:

In the diagram below, a circle is drawn passing through A, B, C and D.

- $B\hat{E}D = \theta$.
- BE and ED are tangents at B and D respectively.

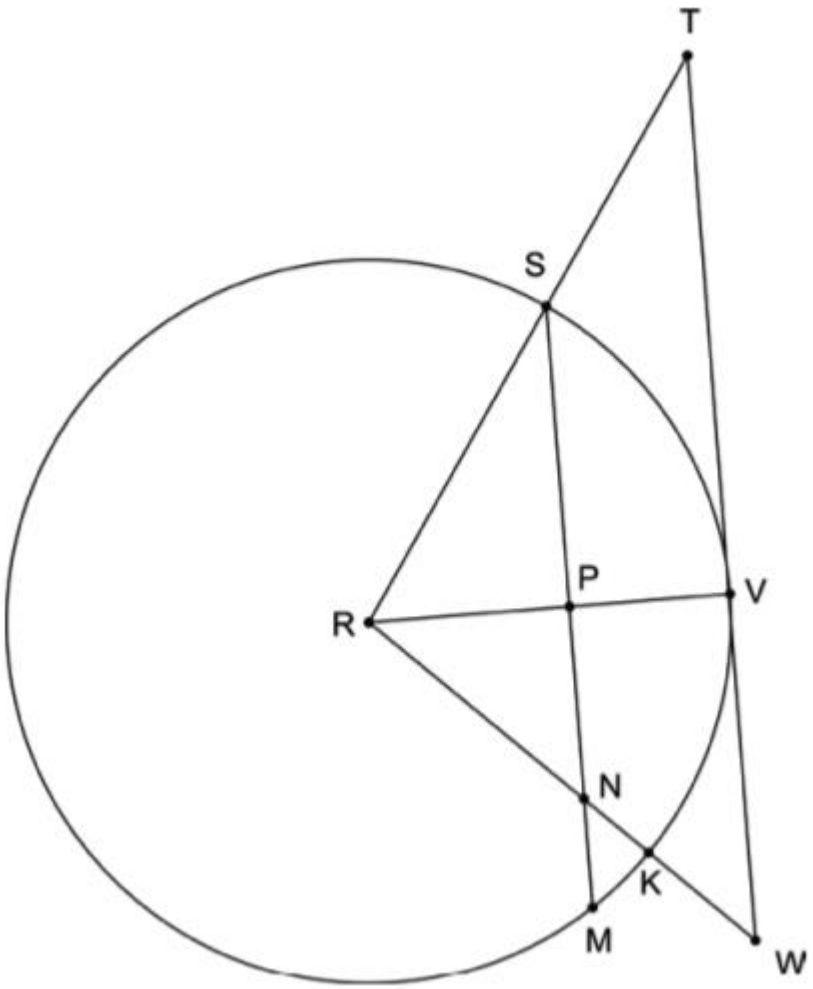


Prove that $B\hat{C}D = 90° + \frac{\theta}{2}$. (6)

QUESTION 13:

In the diagram below:

- TW is a tangent to circle centre R at point V.
- Radius RV intersects chord SM at P and MP = PS.
- The circle has a radius of 10 units.
- RST and RKW are straight lines.
- RW intersects the circle at K and chord SM at N.
- ST = 7 and NW = 6 units.



(a) Prove that TW//SN. (4)

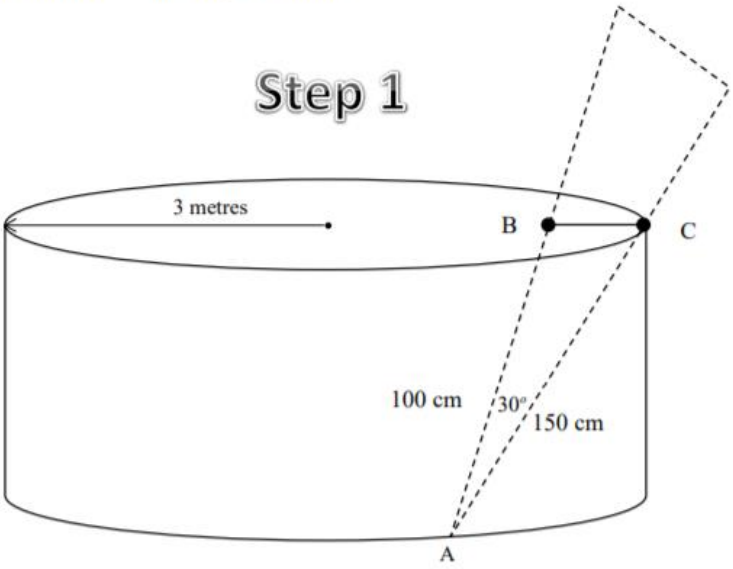
(b) Determine the length of NK. (5)

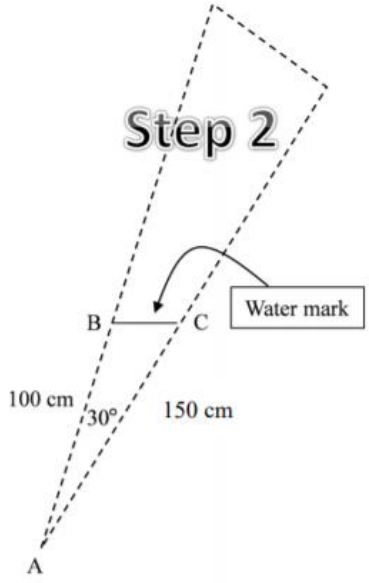
(c) Calculate the length of PN. (8)

QUESTION 14:

A farmer has come up with an accurate way to measure the volume of water in his water tank:

- You place the triangular wedge with $\hat{A} = 30^{\circ}$ in the water tank until point A is touching the bottom of the tank. (Step 1)
- You pull the triangular wedge out of the tank and look for the water mark made on the wedge. This is indicated in Step 2 as line BC.





If the farmer's water tank has a radius of 3 metres, and he measures AB to be 100 cm and AC to be 150 cm, determine the volume of water in the water tank.
(Only round off at the end of your calculation.) $v = \pi r^2 h$

(8)

Activity 6 **15 min**

Now we can share your solutions and discuss aspects of them relating to the problem-solving techniques. (Solutions to the above exam questions can be found in **Appendix 5** in the Resources handout.)

Think about the two sets of problems you have done in the previous activities, one without marks shown and then one with marks. Consider the following:

- How does the mark allocation (or lack thereof) affect you when approaching a problem?
- Should we rely on marks to give us an indication of the level of difficulty of the problem?
- Is there a correlation between level of difficulty and number of marks in the assessments which we give to our learners?
- Are examiners allocating correct marks to problems?

Session 6: The problems with problem-solving at school 30 min

Activity 7 **15 min**

Work in your groups. List the issues you face with problem-solving in your classroom. Try to suggest a solution for each issue.

You could record your ideas in a table like the one below – copy the framework and fill it in.

PROBABLE ISSUES	SOLUTIONS

Activity 8 **15 min**

Give feedback from your group about the issues and suggestions.

After the feedback, refer to **Appendix 2** in the Resources guide for a list of probable issues and solutions and add them to your list if relevant for you.

Session 7: What is the solution? Why do we need more problem-solving in our classrooms? 15 min

Activity 9 **15 min**

General discussion:

- How can we bring problem-solving into our classroom on a regular basis, given the curriculum time constraints?
- How can we help our learners to start to enjoy the challenges of solving problems?
- What are the benefits of bringing more problem-solving to our classrooms?

After this discussion has taken place, refer to Appendix 3 in the Resources handout, for more thoughts and suggestions.

Session 8: Closing 25 min

Activity 10 **20 min**

Post-workshop activity: Re-do the pre-workshop activity. Apply Polya's technique to these questions to determine your solutions.

How would you evaluate this workshop? **5 min**

Respond to this question by writing down your thoughts and **possible suggestions** for future workshops of this nature.

Thank-you for attending today's workshop!
